Optimal FX Interventions with Limited Reserves*

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Abstract

We investigate the optimal time-consistent use of foreign exchange interventions (FXI) in a small open economy model driven by endowment and portfolio flow shocks, with endogenous FX market depth and a lower bound constraint on FX reserves. In a competitive equilibrium, large capital flows increase conditional exchange rate volatility and make FX markets more shallow. Unlike in the unconstrained case, the central bank's optimal interventions are not solely targeted at offsetting inefficient fluctuations in the UIP premium but also incorporate a forward-looking element due to the risk of depleting reserves. We show that this environment leads to optimal time-consistent FXI policy that is highly state-dependent. FX sales are more effective than FX purchases, and the policy may respond less or more than one-for-one to capital outflows, depending on their size and the economy's net foreign assets position. Adopting the policy delivers sizable welfare gains, significantly exceeding those from a simple rule directed at stabilizing current capital outflows, but only if the initial level of FX reserves is sufficiently high.

JEL Codes: E44, F31, F32, F42

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1 Introduction

Volatile international capital flows are a significant concern for policymakers in small open economies, especially in more shock-prone environments amid rising geopolitical tensions, as noted in a recent speech by the IMF's First Deputy Managing Director Gopinath (2024). In shallow FX markets, capital flow volatility exacerbates exchange rate fluctuations, distorting external financing conditions and impeding international risk-sharing. Foreign exchange interventions (FXI) are a key policy tool that can potentially be employed to mitigate these distortive effects, as emphasized by recent open economy literature (see e.g., Basu et al., 2020; Adrian et al., 2022). However, when deciding on the corresponding intervention strategies, policymakers must consider the risk of depleting FX reserves in the future, which brings intertemporal considerations to the forefront. Thus, key questions pertaining to the implementation of FXI policies are about the extent to which portfolio flows should be offset, as well as the average level of FX reserves associated with optimal interventions.

We address these and related questions by studying the use of FXI in a model with endogenous FX market depth and a lower bound on FX reserves, akin to that of Itskhoki & Mukhin (2023). In our framework, which is otherwise a standard small open economy model, FX markets are shallow (i.e., not perfectly elastic) due to the presence of risk-averse agents (dubbed financiers) who are exposed to FX risk when intermediating cross-border capital flows. This market structure gives rise to endogenous deviations from the uncovered interest rate parity (UIP) condition, with FX market depth depending negatively on conditional exchange rate volatility. As a result, when exchange rate uncertainty is high, the economy becomes particularly vulnerable to volatile capital flows, including those stemming from non-fundamental swings in international investors' appetite for domestic currency.

In such an environment, the central bank can use FX interventions to facilitate international risk sharing and to mitigate inefficiencies due to financial intermediation frictions. As an illustration, consider an exogenous fall in the demand for domestic currency, which we refer to as a portfolio capital outflow shock. All else equal, financiers will accommodate the excess supply by buying domestic currency assets, which they will finance by selling foreign currency ones. Since these agents are risk-averse, and since the additional intermediation increases their net long FX exposure to the domestic currency, therefore the UIP premium increases and the domestic currency depreciates, compensating intermediaries for taking on the additional risk. In line with the literature that we build on (Gabaix & Maggiori, 2015; Itskhoki & Mukhin, 2021), FXI is effective since it affects the amount of funds intermediated by financiers. In our capital outflow example, a sterilized FX intervention – i.e., a purchase of domestic currency bonds financed by a sale of foreign currency bonds – lowers the exposure of financiers to the domestic currency, and hence mitigates the movement in the UIP premium.

Indeed, if this intervention could be conducted in an unconstrained fashion, it would be optimal for the central bank to take on the role of financiers, since their intermediation is costly from a social welfare perspective. Consequently, optimal FXI would fully offset portfolio flow shocks and it would eliminate ex-ante UIP deviations, resulting in the best possible allocation achievable, which we will refer to as the "first-best" (FB).[1] In this scenario, while the volatility of the exchange rate due to portfolio flow shocks would be fully eliminated, optimal FXI policy would still allow the exchange rate to fluctuate, facilitating efficient expenditure switching in response to fundamental shocks.

However, if the central bank's ability to sell foreign currency bonds is limited, which is equivalent to its stock of FX reserves being bounded from below, then FX interventions may not be able to fully offset large capital outflows. Importantly, even if the monetary authority was unconstrained at the time, it would have to take into account the fact that its FX reserves may run out under some future scenarios. One consequence is that FX markets become more shallow during episodes of portfolio outflows, as the conditional exchange rate volatility is relatively more affected by non-fundamental forces. The optimal second-best FXI policy also becomes more nuanced, as it is influenced by intertemporal considerations.

The primary contribution of our paper is an analytical and quantitative characterization of optimal time-consistent FXI policy in the simple theoretical setup outlined above. The reason we focus on this type of policy is that, as shown by Itskhoki & Mukhin (2023), the presence of an occasionally binding constraint on reserves makes optimal plans under commitment time-inconsistent.[2] In contrast to that paper, which relies on a linear-quadratic approximation of the equilibrium conditions, we solve the exact nonlinear policy problem using a global and fully nonlinear solution algorithm.

The different approach we take has important consequences for our results. From a positive perspective, our non-linear model can capture time-variation in conditional exchange rate volatility, which increases (falls) whenever the exchange rate depreciates (appreciates). This pattern is consistent with empirical evidence on the behavior of currencies during risk-off episodes (De Bock & de Carvalho Filho, 2015). On the normative side, we demonstrate that optimal time-consistent policy deviates from perfectly stabilizing the UIP risk premium

[1] Our model assumes incomplete financial markets, with risk-free bonds being the only traded assets, which also restricts international risk sharing. In line with the literature, however, we shall assume that policymakers take the structure of international financial markets as given.

[2] Expressed alternatively, such policies are less appealing as – in the absence of an effective commitment device – they lack credibility, with reneging being optimal ex post.

mainly to facilitate intermediation in potentially constrained future states, thus relaxing the implicit borrowing limit faced by the domestic economy. Interestingly, this may imply holding less FX reserves compared to the first-best if capital outflows are relatively mild and expected to abate. However, the focus shifts to maintaining a precautionary level of reserves when outflows intensify, in which case optimal policy chooses not to offset them completely, even if the current level of FX reserves makes full contemporaneous stabilization possible. This precautionary motive crucially depends on endogenously varying FX market depth,[3] which the central bank can influence by "keeping the powder dry", i.e., hoarding FX reserves so that they can be used to lean against future capital outflows. In the case of portfolio inflows, since the lower bound on FX reserves is not a concern, the optimal policy resembles the first-best, in which portfolio capital inflows are matched by buying FX reserves approximately one-for-one.

In the quantitative part of our analysis, we calibrate the model to Malaysia, a small open economy that actively uses FXI to manage its exchange rate. We back out the exogenous process for portfolio flows by applying the UIP condition from the model to the data. Our quantitative results confirm that the optimal time-consistent FXI policy reacts differently to small capital outflows than to large ones. While the theoretical model suggests that the optimal intervention could be more than one-for-one in some circumstances, this case turns out to be quantitatively negligible as the precautionary motive to hoard reserves clearly dominates. We additionally find that – if FX interventions are conducted optimally – then FX purchases tend to affect the real exchange rate by less than FX sales of the same magnitude. This is because purchases occur in periods of portfolio inflows, which are associated with relatively deep FX markets, while sales coincide with portfolio outflows, when FX markets are endogenously shallower.

According to our model, while the standard deviation of the estimated portfolio flow process amounts to 4% of annual GDP, the average level of FX reserves in the optimal time-consistent policy regime is around 5% of GDP. This relatively low level of reserves, coupled with the aforementioned intervention rules, ensures that the unconditional probability of running out of reserves is merely 2%. The policy also reduces the volatility of the exchange rate relative to the no-intervention regime, but it does not fully stabilize it, allowing the exchange rate to induce expenditure switching. Relatedly, FX markets end up significantly deeper when the policymaker intervenes optimally, meaning that the economy is better insulated from inefficient fluctuations brought about by swings in the appetite for domestic currency.

[3] In existing quantitative models (Itskhoki & Mukhin, 2021; Adrian et al., 2022; Davis et al., 2023, and others), FX market depth is constant and typically captured by a calibrated parameter.

Finally, we compute the welfare gains associated with adopting the optimal time-consistent FXI policy, relative to a counterfactual of no intervention, and assuming that the initial FX reserves correspond to their average value historically observed in Malaysia. The gains turn out to be sizable, amounting to 0.25% of permanent steady-state consumption, coming fairly close to those associated with the first-best policy, which establishes their upper bound at 0.29%. These gains are also significantly larger than those implied by a simple static FXI rule, which aims at offsetting current capital flows and hence eliminating the ex ante UIP premium unless FX reserves are fully depleted. However, a significant part of the welfare benefit under the optimal time-consistent policy derives from the gradual decumulation of reserves, as these are initially excessive from the perspective of our model. When these transition gains are controlled for, welfare differences narrow considerably and the optimal time-consistent policy may even be slightly outperformed by commitment to the simple rule responding contemporaneously to capital flows.

Related Literature The theoretical foundation that gives rise to FXI efficacy in open economy models like ours is segmentation in international financial markets. Within a two-country dynamic general equilibrium framework, Devereux & Sutherland (2010) present an approximation method for characterizing time-varying equilibrium portfolios. Gabaix & Maggiori (2015) and Itskhoki & Mukhin (2021) provide microfounded models of a portfolio balance channel in currency markets, where some of the underlying ideas can be traced back to Kouri (1976). Similarly to Cavallino (2019), the role of FXI in our model is to address the associated friction by stabilizing UIP deviations, thus smoothing inefficient fluctuations in external financing conditions. This is consistent with the role played by FXI in models with multiple policy tools such as Itskhoki & Mukhin (2023) and the IMF's Intergrated Policy Framework (Basu et al., 2020; Adrian et al., 2022). Other related studies include Davis et al. (2023), Arce et al. (2019), Chang (2018), and Jeanne & Rancière (2011), who examine the macroprudential use of FXI to prevent sudden stops in emerging economies. Fanelli & Straub (2021) stress the forward guidance component of FX interventions and its time-inconsistency, while Babii et al. (2025) study global consequences of using this policy by a group of countries. Amador et al. (2020) and Cwik & Winter (2024) argue that FXI can function as an effective unconventional monetary policy tool when interest rates are at the effective lower bound (ELB) and when appreciation pressures persist. Bacchetta et al. (2023) complement this literature by analyzing the costs of FXI in economies with safe haven currencies, emphasizing the importance of distinguishing between UIP and CIP deviations.

While arguably highly relevant in policy circles, constraints on FX reserve holdings have attracted only limited attention in the academic literature. One of the few exceptions is

Basu et al. (2018), who examine the optimal use of FXI under limited reserves in a semi-structural setup where the policy objective is assumed to be exchange rate stabilization. Similarly to our work, they show that the lower bound on reserves renders optimal policy time inconsistent, and that simple intervention rules can outperform discretionary policies. A more recent paper developing a structural model closely related to ours is that of Itskhoki & Mukhin (2023). The authors use a linear-quadratic approximation to examine optimal FXI when reserve holdings are constrained, finding that it prescribes more aggressive interventions compared to first-best. In contrast, the adoption of a fully non-linear framework allows us to show that the motive they highlight is typically dominated by precautionary considerations, which imply responding less than one-for-one to capital outflows.

The effectiveness of FXI in our model critically depends on FX market depth (i.e., the elasticity of currency demand), the estimation of which has been a key focus in the empirical literature (Chen et al., 2023; Hertrich & Nathan, 2023; Adler et al., 2019; Fratzscher et al., 2019; Blanchard et al., 2015; Fatum & M. Hutchison, 2003, among others). Major difficulties for these studies stem from limited availability of non-confidential, high-frequency data on foreign exchange interventions, as well as considerable identification issues. Progress has been made in this regard by Adler et al. (2025), who construct a comprehensive set of FXI proxies on a monthly and quarterly basis, which we leverage when calibrating our model. Beltran & He (2024), Pandolfi & Williams (2019) and Broner et al. (2021) exploit exogenous changes in portfolio weights of benchmark indices of local-currency sovereign debt to infer the sensitivity of the exchange rate to capital flows. Maggiori (2022) discusses promising directions in this line of research.

Methodologically, our paper connects to the literature that computes decentralized and constrained-efficient equilibria in small open economies with occasionally binding constraints using global methods (Mendoza, 2010; Bianchi, 2011; Bianchi & Mendoza, 2018; Schmitt-Grohé & Uribe, 2016, 2021; Davis et al., 2023, and others). Similarly to Bianchi & Mendoza (2018), for example, we rely on time iteration of the Euler equation, and we focus on characterizing optimal time-consistent policy.

Outline The remainder of this paper is structured as follows. Section 2 presents the model. The optimal FXI problem is analyzed in Section 3. Section 4 discusses the results of the quantitative analysis and Section 5 evaluates the welfare implications. Finally, Section 6 concludes by summarizing the key findings.

2 Model

This section outlines a standard small open economy model with stochastic endowments, which we extend with a segmented international financial sector that consists of financiers, portfolio investors and the central bank, as in Itskhoki & Mukhin (2023). We first describe the decentralized equilibrium in which the central bank does not engage in FXI, before characterizing the constrained-efficient equilibrium in which the central bank follows a discretionary, time-consistent FXI policy that faces a lower bound on FX reserves.

2.1 Outline

Utility Function Consider an economy that is populated by a large number of identical, infinitely-lived households with preferences described by the following utility function

$$\sum_{t=0}^{\infty} \mathbb{E}_0 \left[\beta^t u\left(C_{T,t}, C_{N,t}\right)\right],$$

where

$$u(C_{T,t}, C_{N,t}) = \frac{1}{1-\sigma}\left(\underbrace{\left[\alpha\left(C_{T,t}\right)^{\frac{\xi-1}{\xi}} + (1-\alpha)\left(C_{N,t}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}}_{C_t}\right)^{1-\sigma}.$$

Households derive utility from total consumption C_t that consists of tradable goods $C_{T,t}$ and nontradable goods $C_{N,t}$. Furthermore, $\beta \in (0,1)$ denotes the subjective discount factor, $1/\sigma$ is the intertemporal elasticity of substitution, ξ is the elasticity of substitution between tradable and nontradable goods, and $\alpha \in (0,1)$ controls the share of tradables in the total consumption basket.

Households' Budget Constraint Each period t, households receive stochastic endowments of tradable and nontradable goods, denoted by $Y_{T,t}$ and $Y_{N,t}$, respectively. The endowments are exogenous and follow a first-order Markov process. Furthermore, households have access to a one period local currency bond B_t that pays gross nominal interest rate R_t. A representative household's sequential budget constraint is then given by

$$P_{T,t}C_{T,t} + P_{N,t}C_{N,t} \leq P_{T,t}Y_{T,t} + P_{N,t}Y_{N,t} - B_t + B_{t-1}R_{t-1} + \Pi_{M,t} + \Pi_{F,t} + \Pi_{P,t}$$

where $\Pi_{M,t}$, $\Pi_{F,t}$ and $\Pi_{P,t}$ denote the profits of the central bank, financiers and portfolio investors, respectively. Note that this formulation of the budget constraint implies that the financial sector (i.e. financiers and portfolio investors) is fully domestically owned.[4]

We assume that the law of one price holds for tradable goods and we normalize the price level abroad to unity, so that the price of tradables equals the nominal exchange rate $P_{T,t} = \mathcal{E}_t$. As we explain below, the price of nontradables is additionally normalized to unity ($P_{N,t} = 1$), so that $\mathcal{E}_t$ can also be interpreted as the relative price of tradable goods.

Households' Optimality The choice of B_t is characterized by the household Euler equation:

$$R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] = 1, \tag{1}$$

where Θ_{t+1} denotes the stochastic discount factor (SDF) of domestic households

$$\Theta_{t+1} = \beta \frac{u_1(C_{T,t+1}, C_{N,t+1})}{u_1(C_{T,t}, C_{N,t})} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\frac{1-\sigma\xi}{\xi}} \left(\frac{C_{T,t+1}}{C_{T,t}} \right)^{-\frac{1}{\xi}}.$$

In addition, combining the first-order conditions with respect to $C_{T,t}$ and $C_{N,t}$ allows us to obtain the equilibrium expenditure switching condition, which pins down the exchange rate

$$\mathcal{E}_t = \frac{u_1\left(C_{T,t}, C_{N,t}\right)}{u_2\left(C_{T,t}, C_{N,t}\right)} = \frac{\alpha}{1-\alpha} \left(\frac{C_{N,t}}{C_{T,t}} \right)^{\frac{1}{\xi}}. \tag{2}$$

Financiers Financiers intermediate funds by holding a zero capital portfolio of foreign currency bonds $B^*_{F,t}$ and local currency bonds $B_{F,t}$ such that $B_{F,t} + \mathcal{E}_t B^*_{F,t} = 0$. They exhibit mean-variance preferences of the form

$$\mathbb{E}_t \left[\Theta_{t+1} \tilde{R}^*_{t+1} B^*_{F,t} \right] - \frac{\omega}{2} var_t \left(\tilde{R}^*_{t+1} B^*_{F,t} \right).$$

Note that the SDF of domestic households Θ_{t+1} enters the objective function of financiers and that $\omega > 0$ measures the (additional) degree of financiers' risk aversion. Moreover, R^* is the (constant) world interest rate and $\tilde{R}^*_{t+1} = R^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ denotes the carry trade return in foreign currency from period t to $t+1$. The first-order condition with respect to $B^*_{F,t}$ yields the following risk-augmented UIP condition

$$R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] - R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = \overbrace{-\omega \sigma_t^2 B^*_{F,t}}^{\text{Risk-Sharing Wedge}}, \tag{3}$$

where $\sigma_t^2 = R_t^2 var_t \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right)$. The ex-ante UIP deviation, defined in Equation 3 as the excess

[4] We discuss the rationale and implications of this assumption in Section 3.

return on domestic currency – which can also be interpreted as the international risk-sharing wedge – is driven by the amount of funds intermediated by financiers $B^*_{F,t}$ and the depth of FX markets $\omega\sigma_t^2 > 0$. The higher the long exposure of financiers to domestic currency assets (i.e., the more negative $B^*_{F,t}$), the higher the compensating excess return they require. Also note that a higher (lower) $\omega\sigma_t^2$ corresponds to shallower (deeper) FX markets, implying greater (lower) sensitivity of the exchange rate to capital flows. Importantly, FX market depth is endogenously state-dependent, with its variation driven by changes in the conditional exchange rate volatility σ_t^2. Finally, the domestic currency profits of financiers in period t are given by $\Pi_{F,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{F,t-1}$.

Portfolio Investors Similar to financiers, portfolio investors hold a zero capital portfolio $(B^*_{P,t}, B_{P,t})$ such that $B_{P,t} + \mathcal{E}_t B^*_{P,t} = 0$. However, they are modeled as non-optimizing agents who randomly buy (sell) foreign currency bonds $B^*_{P,t} > 0$ ($B^*_{P,t} < 0$) and sell (buy) domestic currency bonds $B_{P,t} < 0$ ($B_{P,t} > 0$). More precisely, $B^*_{P,t}$ is exogenous and follows a first-order Markov process. The profits of portfolio investors are $\Pi_{P,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{P,t-1}$.

Central Bank The monetary authority engages in sterilized FXI by adjusting its stock of foreign reserves $B^*_{M,t}$ and sterilization bonds $B_{M,t}$ such that $B_{M,t} + \mathcal{E}_t B^*_{M,t} = 0$. Crucially, the policy is subject to a lower bound constraint on reserves (LBR), which – to fix attention – we set to zero, but which could easily be set at a different level instead

$$B^*_{M,t} \geq 0.$$

The central bank's profits from its bond portfolio are $\Pi_{M,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{M,t-1}$. As already alluded to, we also assume that the central bank's interest rate policy fully stabilizes nontradable goods prices, which allows us to normalize their level to unity.[5]

Bond Market Clearing Overall, market clearing in the domestic bond market requires

$$B_{F,t} + B_t + B_{P,t} + B_{M,t} = 0,$$

with the net foreign asset (NFA) position in foreign currency defined as

$$B^*_t = B^*_{F,t} + B^*_{M,t} + B^*_{P,t}, \tag{4}$$

which implies $B^*_t = B_t/\mathcal{E}_t$ by domestic bond market clearing combined with the balance

[5]This assumption can be understood as capturing the traditional interest rate policy motive arising from nominal rigidities without explicitly modeling them. More specifically, if prices in the nontradable sector were sticky, then monetary policy that perfectly stabilized them would implement the flexible price equilibrium.

sheet equations of the financial sector.

Resource Constraint In equilibrium, consumption of nontradable goods must equal their endowment

$$C_{N,t} = Y_{N,t}. \tag{5}$$

Consolidating the household's budget constraint by exploiting equation (5) and the expression for the profits of the financial sector yields the following economy-wide resource constraint

$$B_t^* - B_{t-1}^* R^* = Y_{T,t} - C_{T,t}. \tag{6}$$

This equation implies that, at the aggregate level, the domestic economy is borrowing in foreign currency at the world interest rate, which, in turn, follows from the assumption of full domestic ownership of the financial sector.

2.2 Decentralized and Constrained-Efficient Equilibrium

Decentralized Equilibrium Having outlined the model, we now define the decentralized equilibrium, in which the central bank does not conduct FX interventions and does not hold any FX reserves.

Definition 1 (Decentralized Equilibrium without FX Interventions)**.** Given exogenous process $\{B_{P,t}^*, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and initial condition B_{-1}^*, a competitive equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied conditional exchange rate volatility $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$, bond positions $\{B_t^*, B_{F,t}^*\}_{t=0}^{\infty}$, and FXI policy $\{B_{M,t}^*\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize, implying (1), (2), and (3)
2. The central bank holds no FX reserves, $\forall t : B_{M,t}^* = 0$
3. Goods and bond markets clear, implying (5), (6), and (4)
4. The transversality condition on net foreign assets holds

$$\lim_{T \to \infty} \frac{B_T^*}{(R^*)^T} = 0.$$

Constrained-Efficient Equilibrium We next describe the problem of a central bank that optimally conducts FX interventions and we define the associated equilibrium. We assume that every period t the central bank chooses FX reserves $B_{M,t}^*$ in a discretionary fashion, unable to credibly commit to future actions. At the same time, the intervention policy is fully time-consistent as the effects of current actions on future optimal decisions are entirely accounted for. Consequently, the policy problem can be formulated as follows

$$\max_{\{C_{T,t},B_t^*,\mathcal{E}_t,R_t,B_{M,t}^*,\sigma_t^2\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[u(C_{T,t+s}, Y_{N,t+s})\right] \tag{7}$$

subject to the constraints

$$R_{t+s}\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right] = 1 \qquad \text{(Household Euler Equation)}$$

$$\mathcal{E}_{t+s} = \frac{u_1(C_{T,t+s}, Y_{N,t+s})}{u_2(C_{T,t+s}, Y_{N,t+s})} \qquad \text{(Expenditure Switching)}$$

$$B_{t+s}^* - B_{t+s-1}^* R^* = Y_{T,t+s} - C_{T,t+s} \qquad \text{(Resource Constraint)}$$

$$R^*\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right] = 1 + \omega\sigma_{t+s}^2\left(B_{t+s}^* - B_{M,t+s}^* - B_{P,t+s}^*\right) \qquad \text{(International Risk Sharing)}$$

$$\sigma_{t+s}^2 = R_{t+s}^2 var_{t+s}\left(\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right) \qquad \text{(Cond. Exchange Rate Volatility)}$$

$$B_{M,t+s}^* \geq 0 \qquad \text{(Constraint on Reserves)}$$

This allows us to arrive at the following definition of a constrained-efficient equilibrium.

Definition 2 (Constrained-Efficient Equilibrium with FX Interventions)**.** Given the exogenous process $\{B_{P,t}^*, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and initial condition B_{-1}^*, a constrained-efficient equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied conditional exchange rate volatility $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$, bond positions $\{B_t^*, B_{F,t}^*\}_{t=0}^{\infty}$ and FXI policy $\{B_{M,t}^*\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize, implying (1), (2), and (3)
2. The central bank solves the policy problem (7)
3. Goods and bond markets clear, implying (5), (6), and (4)
4. Transversality condition on net foreign assets holds

$$\lim_{T\to\infty} \frac{B_T^*}{(R^*)^T} = 0.$$

3 Optimal FX Interventions

We proceed by analytically studying the optimal time-consistent use of FXI in the model described in Section 2. We first characterize the first-best and next describe the key results obtained from the analysis of the second-best policy. The proofs of all theorems and propositions presented in this section can be found in Appendix A.

First-best If households had direct, frictionless access to foreign currency bonds, the first-order condition associated with that asset would be

$$R^* \mathbb{E}_t \left[\Theta_{t+1}\right] = 1. \tag{8}$$

It is straightforward to show that this condition holds in the constrained-efficient equilibrium without an LBR as long as the financial sector is entirely owned by domestic households, which implies that the economy effectively borrows at the foreign interest rate. The associated optimal FXI policy is characterized by

$$B^*_{M,t} = B^*_t - B^*_{P,t}, \tag{9}$$

which fully eliminates the international risk-sharing wedge. In particular, the central bank responds one-for-one to fluctuations in the demand for domestic currency, making costly intermediation provided by financiers redundant ($B^*_{F,t} = 0$).[6]

It should be noted that strict stabilization of the UIP premium would no longer be optimal if we allowed financial market participants to be at least partially foreign owned. In that case, the central bank would lean against capital flows less than one-for-one, making systematic profits on FX reserve management at the expense of agents living in the rest of the world (see also Itskhoki & Mukhin, 2023; Adrian et al., 2022, for a more comprehensive discussion).

Financial Conditions under second-best In the presence of an LBR, the central bank is generally unable to completely eliminate the risk-sharing wedge in all states, in contrast to first-best. One can easily imagine a situation in which the economy holds a relatively large amount of debt, or faces a sizable portfolio outflow, such that $B_t - B^*_{P,t} < 0$ and the LBR binds. Before we fully characterize the optimal FXI policy in such circumstances, it is useful to spell out two key implications of this equilibrium for the financial conditions faced by the small open economy. These are summarized in the following Proposition:

Proposition 1. *Consider an equilibrium in period t with a binding LBR ($B^*_t - B^*_{P,t} < 0$) and where a portfolio outflow is associated with an improvement in the net foreign asset position ($\partial B^*_t / \partial B^*_{P,t} > 0$). Then, the equilibrium exhibits the following properties:*

(a) $\partial \sigma^2_t / \partial B^*_{P,t} > 0$: A portfolio outflow leads to an elevated conditional volatility of the

[6] A corollary is that the first-best in an economy affected solely by portfolio shocks is associated with no exchange rate uncertainty, i.e., it corresponds to a peg with $\sigma^2_t = 0$.

exchange rate.

(b) $\partial \mathbb{E}_t [\Theta_{t+1}] / \partial B^*_{P,t} < 0$*: A portfolio outflow leads to an expected decrease of the stochastic discount factor between periods* t *and* $t+1$*.*

The crucial relationship underpinning Proposition 1 is that we consider an economy in which portfolio outflows generate an improvement in the net foreign asset position. While this appears to be the empirically relevant case, we also characterize sufficient conditions for this relationship to hold in Appendix A, which additionally contains a proof of Proposition 1. Intuitively, if the LBR is binding, a portfolio outflow cannot be fully offset by selling FX reserves. Then, by the bond market clearing condition (4), for the NFA position of the economy not to increase, all of the resulting imbalance would have to be absorbed by financiers. However, that implies a tightening of the financial conditions, since financiers require a higher premium when their domestic currency exposure increases. This in turn incentivizes households to increase their savings and it translates into an improvement in the country's net foreign asset position.

Under the conditions specified in Proposition 1, state-dependent FX market depth $\omega\sigma_t^2$ magnifies the impact of portfolio outflows on the risk sharing wedge. Intuitively, a portfolio outflow that cannot be fully offset by an FX intervention causes a positive UIP deviation not only through its impact on the amount of funds that must be intermediated by the financiers, but also by increasing the likelihood that an outflow in the following period would not be fully offset either. The latter implies an endogenous increase in the conditional volatility of the exchange rate, meaning higher riskiness of the financiers' balance sheets, for which they have to be compensated. The associated higher UIP wedge tightens households' financial conditions, as they can only borrow at a greater premium over the foreign interest rate. These tighter conditions thus decrease consumption of tradable goods, and so imply a higher marginal utility today compared to tomorrow. As a result, the stochastic discount factor decreases.

Implicit Borrowing Limit One interesting consequence of the lower bound on FX reserves is that it imposes an upper bound on the country's net foreign liabilities, thus constituting an implicit *borrowing* limit that may constrain optimal policy. To see this, combine the international risk sharing condition (3) with the LBR to obtain

$$B^*_t \geq B^*_{P,t} - \frac{1}{\omega\sigma_t^2}\left(1 - R^*\mathbb{E}_t[\Theta_{t+1}]\right) \equiv \Psi_t. \tag{10}$$

We can hence reformulate the second-best in terms of a consumption smoothing problem subject to this additional constraint.

The borrowing limit Ψ_t has two components: The first one is represented by exogenous portfolio outflows $B^*_{P,t}$, which increase Ψ_t and thus make the borrowing limit tighter. The second term corresponds to the financiers' domestic currency lending position (expressed in foreign currency), and it decreases Ψ_t whenever the LBR is binding, i.e., when $R^*\mathbb{E}_t\left[\Theta_{t+1}\right] < 1$. Due to their mean-variance preferences, the financiers' position in equilibrium is a function of the expected excess return on domestic currency $1 - R^*\mathbb{E}_t\left[\Theta_{t+1}\right]$ and the risk factor $\omega\sigma_t^2$. If either the expected return increases and/or the risk factor decreases, financiers are willing to lend more in domestic currency. In so doing, they relax the implicit borrowing limit affecting optimal policy. Furthermore, optimal FX reserves simply reflect the difference between net foreign assets and the borrowing limit, i.e., $B^*_{M,t} = B^*_t - \Psi_t$. In particular, the borrowing limit is binding ($B^*_t = \Psi_t$) if and only if the central bank runs out of reserves ($B^*_{M,t} = 0$).

Value of Commitment and Time Inconsistency of Optimal Plans One important feature of the implicit borrowing limit given by equation (10) is that it is forward-looking. More specifically, it depends on the conditional volatility of the exchange rate and on the stochastic discount factor that households use to evaluate future flows, both of which depend on expectations formulated at time t about events at time $t+1$. If the central bank could credibly commit to future FX interventions, it could steer these expectations in a way that would relax the current-period borrowing limit whenever it became binding. Such a policy would thus resemble "forward guidance" about the future path of interest rates, which, in a New Keynesian setup, can effectively mitigate the consequences of the effective lower bound (ELB) on the policy rate.

However, and as in the New Keynesian ELB case, committing to future interventions is not time-consistent: once new shocks materialize, it is optimal for the central bank to reoptimize, possibly reneging on past promises. For this reason, in the remainder of our analysis, we mainly focus on fully time-consistent but discretionary FX intervention policies.

Intertemporal Tradeoffs under second-best The first order condition associated with the second-best policy problem is given by

$$
\begin{aligned}
u_{1,t} = {} & \beta R^* \mathbb{E}_t \left[u_{1,t+1} \right] && (FB) \\
& + \lambda_t \left(1 - \frac{1 - R^* \mathbb{E}_t \left[\Theta_{t+1} \right]}{\omega (\sigma_t^2)^2} \frac{\partial \sigma_t^2}{\partial B_t^*} - \frac{R^*}{\omega \sigma_t^2} \mathbb{E}_t \left[\frac{\partial \Theta_{t+1}}{\partial B_t^*} \right] \right) && (\geq 0) \\
& - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{1 - R^* \mathbb{E}_{t+1} \left[\Theta_{t+2} \right]}{\omega \left(\sigma_{t+1}^2 \right)^2} \frac{\partial \sigma_{t+1}^2}{\partial B_t^*} \right] && (\geq 0) \\
& - \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R^*}{\omega \sigma_{t+1}^2} \mathbb{E}_{t+1} \left[\frac{\partial \Theta_{t+2}}{\partial B_t^*} \right] \right] && (\leq 0)
\end{aligned}
\tag{11}
$$

where $u_{1,t} \equiv u_1 \left(C_{T,t}, Y_{N,t} \right)$, and where λ_t denotes the Lagrange multiplier associated with the implicit borrowing constraint (10).[7]

Several observations are in order. First, if the lower bound on reserves is never binding ($\forall t, \lambda_t \neq 0$), then the optimality condition (11) reduces to the first line and becomes equivalent to the first-best characterized by equation (8). An implication is that under such circumstances the central bank would be able to perfectly eliminate the international risk sharing wedge. The second line in equation (11) corresponds to the case of insufficient FX reserves today. An economy that hits the LBR at time t (so that $\lambda_t > 0$) experiences a tightening in financial conditions (UIP premium becomes positive), is forced to borrow less, and hence needs to restrict its consumption. Finally, the last two lines of equation (11) illustrate how the possibility of the LBR becoming binding in the future ($\lambda_{t+1} > 0$ in some states) affects the allocations chosen by the optimizing central bank today, even if the current level of FX reserves is positive. These forward-looking motives of the optimal FXI policy reflect the fact that the central bank internalizes the effects of the economy's current savings on the future financial conditions arising from the presence of the implicit borrowing constraint.

Interestingly, there are two opposing forces at play here. The term in the third line arises since the economy's savings decisions in period t affect the future conditional exchange rate volatility σ_{t+1}^2. While each individual household takes this risk factor as given, the planner internalizes how conditional volatility is affected by economy-wide savings. It can be shown that $\frac{\partial \sigma_{t+1}^2}{\partial B_t^*} < 0$, which means that if the economy saves more today, then the future conditional exchange rate volatility decreases. This is because higher current net foreign assets imply that lower FX interventions are required to prevent an increase in the international risk-sharing wedge for a given portfolio capital outflow. As a result, the probability that any outflow will not be fully neutralized due to insufficient FX reserves decreases, which implies a lower expected impact of non-fundamental shocks on next period's exchange rate. Since

[7] The complementary slackness conditions are $\lambda_t \geq 0,\ B_t^* - \Psi_t \geq 0,\ (B_t^* - \Psi_t)\lambda_t = 0$.

$1 - R^* \mathbb{E}_t [\Theta_{t+2}] > 0$ whenever FXI policy is constrained in period $t+1$ ($\lambda_{t+1} > 0$), the third line of equation (11) is positive, meaning that saving more today brings the benefit of a less binding implicit borrowing limit tomorrow, due to compressed conditional exchange rate volatility.

On the other hand, the last line in equation (11) captures the effect of aggregate savings on the household's period $t+1$ stochastic discount factor, which financiers use to weigh their future profits. Again, an individual household does not take into account the effect of its intertemporal decisions on Θ_{t+2}. However, the central bank internalizes the fact that higher savings in period t support tradable consumption in the future, thus increasing the aggregate stochastic discount factor in the following period. Accordingly, $\frac{\partial \Theta_{t+2}}{\partial B_t^*} > 0$. Note that higher Θ_{t+2} decreases the expected excess return on domestic currency $1 - R^* \mathbb{E}_t [\Theta_{t+2}]$, and hence financiers' expected profits, hindering their capacity to intermediate. As a result, the fourth line of equation (11) is negative, highlighting the fact that saving more today can indirectly tighten the implicit borrowing limit next period by compressing financiers' expected profits and thus amplifying the associated intermediation friction.

Optimal Time-Consistent FXI under second-best We have seen above that the optimizing central bank internalizes two opposing effects of aggregate savings — represented by the economy's net foreign assets position – on the implicit borrowing limit in the next period. The optimal FXI policy in period t is guided by whichever of the two motives dominates, as summarized in the following theorem

Theorem 1. *If the use of FXI is unconstrained in period t ($\lambda_t = 0$) but the LBR is possibly binding in period $t+1$ ($\mathbb{E}_t [\lambda_{t+1}] > 0$), then the optimal time-consistent FXI policy in period t is given by*

$$B_{M,t}^* = B_t^* - B_{P,t}^* + \frac{\beta R^*}{\omega \sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right], \tag{12}$$

which calls for a smaller (larger) intervention relative to the first-best if the third term on the RHS is positive (negative).

Theorem 1 states that the optimal intervention in period t in anticipation of a binding LBR in period $t+1$ can either be liquidity-injecting, in the sense that the central bank decides to hold less reserves compared to the first-best ($B_{M,t}^* < B_t^* - B_{P,t}^*$), or liquidity-absorbing to the extent that the central bank stocks more reserves ($B_{M,t}^* > B_t^* - B_{P,t}^*$).[8] If the LBR

[8]The level of FX reserves directly influences liquidity in domestic bond markets, as interventions are conducted in a sterilized manner. Compared to the first-best, the central bank is a net buyer of domestic bonds in the former case and a net seller in the latter case.

is expected not to bind in any possible states in period $t+1$, the optimal intervention in period t corresponds to the first-best.

To grasp the intuition behind Theorem 1, recall that the risk sharing wedge provides a read on the severity of the intermediation friction in our model. While it can always be completely eliminated in the first-best, this may not be feasible in the second-best case. By definition, a period $t+1$ characterized by a binding LBR is associated with negative intermediated funds $B^*_{F,t+1} = B^*_{t+1} - B^*_{P,t+1} < 0$ and consequently also a positive risk sharing wedge $-\omega\sigma^2_{t+1}B^*_{F,t+1} > 0$. The aim of the second-best policy in this context is to relax the implicit borrowing limit by facilitating intermediation by financiers. Recall that the intermediated funds at time $t+1$ depend positively on the expected return $1 - R^*\mathbb{E}_{t+1}[\Theta_{t+2}]$ and negatively on the risk factor $\omega\sigma^2_{t+1}$. The central bank can affect both of these variables by engaging in FXI in period t. A liquidity-injecting intervention $B^*_{M,t} < B^*_t - B^*_{P,t}$ can increase the expected return $1 - R^*\mathbb{E}_{t+1}[\Theta_{t+2}]$ and thereby increase financiers' capacity to lend to domestic households in the possibly constrained period $t+1$. Conversely, a liquidity-absorbing intervention $B^*_{M,t} > B^*_t - B^*_{P,t}$ can decrease the conditional volatility of the exchange rate σ^2_{t+1}, making financiers more willing to lend.

Precautionary Accumulation of FX Reserves The presence of a liquidity-absorbing intervention motive is the key difference between our analysis and that recently offered by Itskhoki & Mukhin (2023). In contrast to our global and nonlinear approach to the optimal policy problem, their analytical framework relies on a novel, first-order approximation to the equilibrium system. The latter brings substantial gains in terms of tractability, but it effectively assumes away the liquidity-absorbing motive, only allowing for the liquidity-injecting deviation from first-best. As a consequence, if the central bank is unconstrained in period t, but risks running out of FX reserves in subsequent periods, their model implies intervening more than one-for-one to a capital outflow, resulting in a *negative* UIP risk premium.[9]

The liquidity-absorbing intervention motive captured by our analysis works in the opposite direction and has a precautionary flavor. It relies on the negative impact of conditional exchange rate volatility on FX market depth. Specifically, any given portfolio outflow shock has less of an effect on the risk sharing wedge in the constrained period $t+1$ if FX markets are deeper, which happens when the conditional exchange rate volatility σ^2_{t+1} is lower. Crucially, as discussed above, entering the constrained period with larger net foreign assets reduces conditional exchange rate volatility, which – in contrast to households – the central bank internalizes. By accumulating reserves in period t beyond what the first-best suggests, the

[9] See Theorem 2 in Itskhoki & Mukhin (2023).

central bank forces the whole economy to save more. This, in turn, makes it less likely that a given portfolio capital outflow in period $t+1$ will lead to the depletion of FX reserves, which decreases the loading of non-fundamental forces on exchange rate risk. In general, a liquidity-absorbing intervention $B^*_{M,t} > B^*_t - B^*_{P,t}$ can be interpreted as "keeping powder dry", as it results in the precautionary accumulation of FX reserves. Accordingly, by engaging in such interventions, the central bank can smooth the international risk-sharing wedge over time by compressing the UIP premium in the potentially constrained period $t+1$, at the cost of increasing it contemporaneously in the unconstrained period t.

Conditions under which the liquidity-absorbing motive dominates the liquidity-injecting motive are characterized in the following proposition.

Proposition 2. *Suppose the use of FXI is unconstrained in period t ($\lambda_t = 0$) but the LBR is possibly binding in period $t+1$ ($\mathbb{E}_t[\lambda_{t+1}] > 0$). Then the optimal level of reserves is higher relative to first-best if and only if*

$$R^*\mathbb{E}_t[\Theta_{t+2}] < \frac{2}{3} - \frac{\mathbb{C}ov_t\left(\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right), R^*\mathbb{E}_{t+1}[\Theta_{t+2}]\right)}{\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\right]}, \tag{13}$$

which is more likely to hold if the (expected) stochastic discount factor between the potentially constrained period $t+1$ and period $t+2$ is lower.

According to Proposition 2, the liquidity-absorbing motive prevails if the stochastic discount factor is expected to drop sufficiently in the possibly constrained period $t+1$. In other words, if households are expected to experience a particularly severe drop in tradable consumption when the economy hits the implicit borrowing limit (the LBR becomes binding), then it is optimal for the central bank to hold more reserves than in the first-best (i.e., to absorb liquidity). By doing so, policymakers can make the potential crisis less severe if it unfolds.

The natural question is what type of potential crisis would justify holding more reserves than in the first-best as a form of precaution. First, equation (12) indicates that higher portfolio outflows $B^*_{P,t+1}$ strengthen the incentive to accumulate reserves at time t. Larger outflows in period $t+1$ raise the UIP premium, tightening external financing conditions and forcing households to reduce consumption. This effect is amplified when the economy already carries a substantial debt burden.

This leads to the second point: the NFA position in period $t+1$ can be written as $B^*_{t+1} = R^*B^*_t + TB_{t+1}$, where $TB_{t+1} = Y_{T,t+1} - C_{t+1}$ denotes the trade balance. A stronger NFA position in period t lowers the likelihood of a sharp decline in consumption in period $t+1$, and thus reduces the need to hold precautionary reserves in period t. Finally, a lower tradable endowment $Y_{T,t+1}$ reduces the trade balance TB_{t+1}, reinforcing the case for holding more reserves at time t. The relevance and interaction of these channels are best analyzed in a quantitative setting, which is the focus of the next section.

4 Quantitative Analysis

We begin by describing the calibration of the model presented above and then analyze its quantitative implications, using remarks to highlight the key insights.

4.1 Calibration

We calibrate the model to Malaysia, a small open emerging economy with a central bank that is an active user of FXI. We think Malaysia is a fitting case to explore through the lens of our model since its central bank uses FXI continuously to stabilize the exchange rate, especially in periods of large and volatile capital flows (Aziz, 2019). More specifically, we calibrate the model to data spanning 2010 to 2023, when the Malaysian ringgit operated under a managed float regime and Bank Negara Malaysia (BNM) conducted FX interventions whenever "ringgit market movements were not orderly and to ensure enough liquidity in the banking system" (BNM, 2025). While this period was admittedly characterized by relatively stable economic conditions and no major domestic financial crisis, intervention estimates suggest that the central bank was active in FX markets in both directions.[10]

Table 1: Calibration

Description	Value	Source/Target
World interest rate, quarterly	$R^* = 1.01$	Standard value DSGE-SOE
Relative risk aversion	$\sigma = 2$	Standard value DSGE-SOE
Elasticity of substitution of T-NT goods	$\xi = 0.83$	Bianchi (2011)
Weight on traded goods in CES aggregator	$\alpha = 0.39$	Malaysia's economy
Subjective discount factor, quarterly	$\beta = 0.9871$	NFA-GDP ratio, Malaysia's economy
Financiers' risk aversion	$\omega = 28$	FX market depth, Davis et al. (2023)

The parameter values of the model are listed in Table 1. We calibrate the model at a quarterly frequency. The annualized world interest rate is 4% and the relative risk aversion σ is set

[10]Based on estimates from Adler et al. (2025), the standard deviation of quarterly spot FX interventions by the BNM between 2010 and 2023 amounted to 1.5% of annual GDP. The frequencies of interventions in both directions were similar, with 27 quarters of net purchases and 29 quarters of net sales.

to 2, both standard in the small open economy literature. Regarding the parameter guiding the elasticity of substitution between tradable and nontradable goods ξ, we follow Bianchi (2011) and choose a value of 0.83 that is at the upper bound of the empirical estimates. We calibrate the weight on traded goods in the CES aggregator α equal to the average share of the tradable component in Malaysian GDP, which is 39%. Since Malaysia is a frequent user of FXI, we need to take its FXI regime into account when calibrating β and ω.[11] Our chosen value of the subjective discount factor β targets the Malaysian private NFA-to-GDP ratio, while the financiers' risk aversion ω is set to imply an average FX market depth $\omega\overline{\sigma}^2 = 0.05$, consistent with Davis et al. (2023). While Appendix B provides additional details on this part of the calibration, we note that our chosen values of β and ω imply a reasonable reaction of the exchange rate to capital flows, including FX interventions.[12]

For the endowment part of the exogenous driving forces in our model $\{Y_{T,t}, Y_{N,t}\}$, we follow the standard methodology in the literature and use the cyclical components of tradable and nontradable GDP at constant prices, retrieved from BNM statistics. We classify agriculture, mining and quarrying, and manufacturing as tradables and we treat the rest of GDP as nontradable. To obtain the cyclical components, we remove a cubic trend and seasonality from the natural logarithm of tradables and nontradables, in line with Schmitt-Grohé & Uribe (2016).

To estimate a process for exogenous portfolio outflows $\{B^*_{P,t}\}$, we first calculate an empirical measure of the risk sharing wedge (or ex-ante UIP deviation) as defined in our model, i.e.,

$$\widehat{RSW}_t = \widehat{\Theta}_{t+1} \left(\hat{R}^*_t - \hat{R}_t \frac{\hat{\mathcal{E}}_t}{\widehat{\mathbb{E}_t \left[\mathcal{E}_{t+1} \right]}} \right), \tag{14}$$

where $\hat{R}^*_t$ is the effective federal funds rate, $\hat{R}_t$ denotes the BNM overnight policy rate, $\hat{\mathcal{E}}_t$ is the USD to Malaysian Ringgit (MYR) spot rate, and $\widehat{\mathbb{E}_t \left[\mathcal{E}_{t+1} \right]}$ is the Bloomberg composite one-quarter ahead forecast of the USD/MYR exchange rate. Furthermore, our measure of the stochastic discount factor is given by $\widehat{\Theta}_{t+1} = \beta \left(\hat{C}_{t+1}/\hat{C}_t \right)^{\frac{1-\sigma\xi}{\xi}} \left(\hat{C}_{T,t+1}/\hat{C}_{T,t} \right)^{-\frac{1}{\xi}}$ where we compute $\hat{C}_t$ and $\hat{C}_{T,t}$ using the cyclical components of tradable and nontradable GDP (see above) as well as the quarterly trade surplus retrieved from the IMF IFS.[13] Next, we make

[11]Given our calibration strategy, both β and ω crucially depend on the FXI regime because a higher level of reserves increases the model net foreign asset position, while an FXI regime that dampens real exchange rate volatility is associated with deeper FX markets.

[12]More specifically, if we simulate the data from our baseline model and regress the log change in the exchange rate on portfolio outflows (net of FXI and expressed as a percentage of annual GDP), controlling for endowment shocks, we obtain a coefficient of around 0.6. This number increases to 1.3 if we use data from the decentralized equilibrium (without FXI) but drops to 0.4 under optimal time-consistent policy.

[13]Note that the SDF $\widehat{\Theta}_{t+1}$ in (14) is computed using realized consumption in period $t + 1$, whereas the

use of the international risk-sharing condition to back out exogenous portfolio outflows

$$\hat{B}^*_{P,t} = \hat{B}^*_t - \hat{B}^*_{M,t} - \frac{\widehat{RSW}_t}{\omega \hat{\sigma}^2_t}, \tag{15}$$

where we obtain the quarterly measure of Malaysia's NFA position $\hat{B}^*_t$ from the IMF IFS, the quarterly proxy for Malaysia's FXI from Adler et al. (2025), and where we use the three-month implied USD/MYR volatility from Bloomberg to retrieve $\hat{\sigma}^2_t$.[14]

We additionally model exogenous states as a first-order vector autoregression $\mathbf{s}_t = \rho \mathbf{s}_{t-1} + \varepsilon_t$, where $\mathbf{s}_t = \left[\log Y_{T,t}, \log Y_{N,t}, \sinh^{-1}\left(B_{P,t} - \overline{B}_P\right)\right]'$ are obtained as described above for the period 2010:Q1 to 2023:Q4.[15] The error term $\varepsilon_t = [\varepsilon_{T,t}, \varepsilon_{N,t}, \varepsilon_{P,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\mathbf{V}$, while ρ is a 3×3 matrix comprising first-order autocorrelation terms:

$$\mathbf{V} = \begin{bmatrix} 0.0005447 & 0.0005911 & 0.0019075 \\ 0.0005911 & 0.0008851 & 0.0013138 \\ 0.0019075 & 0.0013138 & 0.1727534 \end{bmatrix}, \quad \rho = \begin{bmatrix} 0.8213977 & -0.3171368 & -0.0201376 \\ 0.2110661 & 0.3794069 & -0.0260989 \\ -0.650205 & -0.1477713 & 0.4799129 \end{bmatrix}.$$

Without loss of generality, we normalize the mean of the endowment processes to one, while the mean of portfolio outflows equals $\bar{B}_P = -3.09$, indicating that Malaysia experiences portfolio inflows on average. Furthermore, the unconditional standard deviations of the exogenous driving forces are $\sigma_{Y_T} = 0.029$, $\sigma_{Y_N} = 0.037$ and $\sigma_{B^*_P} = 0.521$. The tradable and nontradable endowment processes are highly positively correlated ($\sigma_{Y_T,Y_N} = 0.795$) while portfolio outflows are weakly negatively correlated with endowments ($\sigma_{B^*_P,Y_T} = -0.036$ and $\sigma_{B^*_P,Y_N} = -0.147$). This weak correlation between portfolio outflows and the endowment process is a manifestation of the well-documented exchange rate disconnect, i.e., the fact that exchange rates are to a large extent driven by factors other than fundamentals.

The exogenous state variables S are discretized into a first-order Markov process with four grid points for both of the endowment processes $\{Y_T, Y_N\}$, eight grid points for the portfolio flow process B^*_P, and with 1000 grid points for the endogenous state variable B^*.[16] Finally,

SDF in (3) is expressed in terms of expected consumption for that period. We adopt this simplification because data on expected (tradable) consumption is unavailable.

[14]Since Adler et al. (2025) only provide FX interventions data, we additionally use IMF IFS data to account for the level of FX reserves needed to compute $\hat{B}^*_{M,t}$.

[15]Note that we remove the mean $\overline{B}_P$ from the portfolio flow process and apply the inverse hyperbolic sine transformation due to negative values.

[16]Specifically, the grid points are: $Y_T \in \{0.96, 0.99, 1.01, 1.04\}$, $Y_N \in \{0.95, 0.98, 1.02, 1.06\}$, and $B^*_P \in \{-3.81, -3.58, -3.38, -3.18, -2.99, -2.79, -2.59, -2.36\}$.

we solve the decentralized equilibrium and the constrained planner's problem using time iteration on the Euler equation.

4.2 Policy Functions

Figure 1 shows the policy functions for the small open economy's NFA position B^*, separately for the decentralized equilibrium without FXI (blue solid line) and the constrained planner's solution (i.e., optimal FXI; red dashed line). For low levels of the current net foreign asset position, the economy increases its NFA next period, while the opposite is true for relatively high current net foreign asset holdings. As a result, both lines intersect the 45 degree line indicating the existence of stationary equilibria. Intuitively, the financiers' risk aversion gives rise to an upward-sloping supply of funds to the domestic economy, which limits the amount of external borrowing. In addition, Figure 1 reveals that the constrained planner's policy function lies above that characterizing the decentralized equilibrium, highlighting the importance of FX interventions for the economy's steady state NFA position.

To further shed light on the characteristics of the optimal FXI policy, Figure 2 shows the optimal level of FX reserves, B_M^*, as a function of portfolio outflows, B_P^*, distinguishing between states with a relatively low (solid blue) and high (dashed red) NFA position and an average across all NFA states (green dotted). To put these in perspective, the corresponding first-best policies are indicated by the black solid lines. For a relatively high NFA position, optimal policy offsets portfolio outflows approximately one-for-one and deviations from the first-best policy are minor. This is because a relatively high NFA position is associated with ample FX reserves, which decreases the relevance of the lower bound. In contrast, the lower bound on reserves becomes relevant if the economy finds itself in a state with relatively low NFA, in which case the central bank optimally exhausts its reserves in response to sufficiently large portfolio outflows. While optimal reserve holdings may exceed or fall short of the first-best level depending on the state, precautionary considerations dominate on average. This is illustrated by the positive gap between the green dotted line and the corresponding first-best policy across portfolio outflow levels. It is important to emphasize that the flatter section of the average policy function does not simply reflect reserve depletion in some states. Rather, it is driven by the precautionary motive.[17] This leads to our first remark, which connects back to the qualitative analysis in Section 3.

Remark 1. Under the optimal time-consistent policy, reserve holdings may lie above or below

[17] This is confirmed by excluding states with zero reserves from the average, which turns out to have a relatively minor impact on the shape of the policy function.

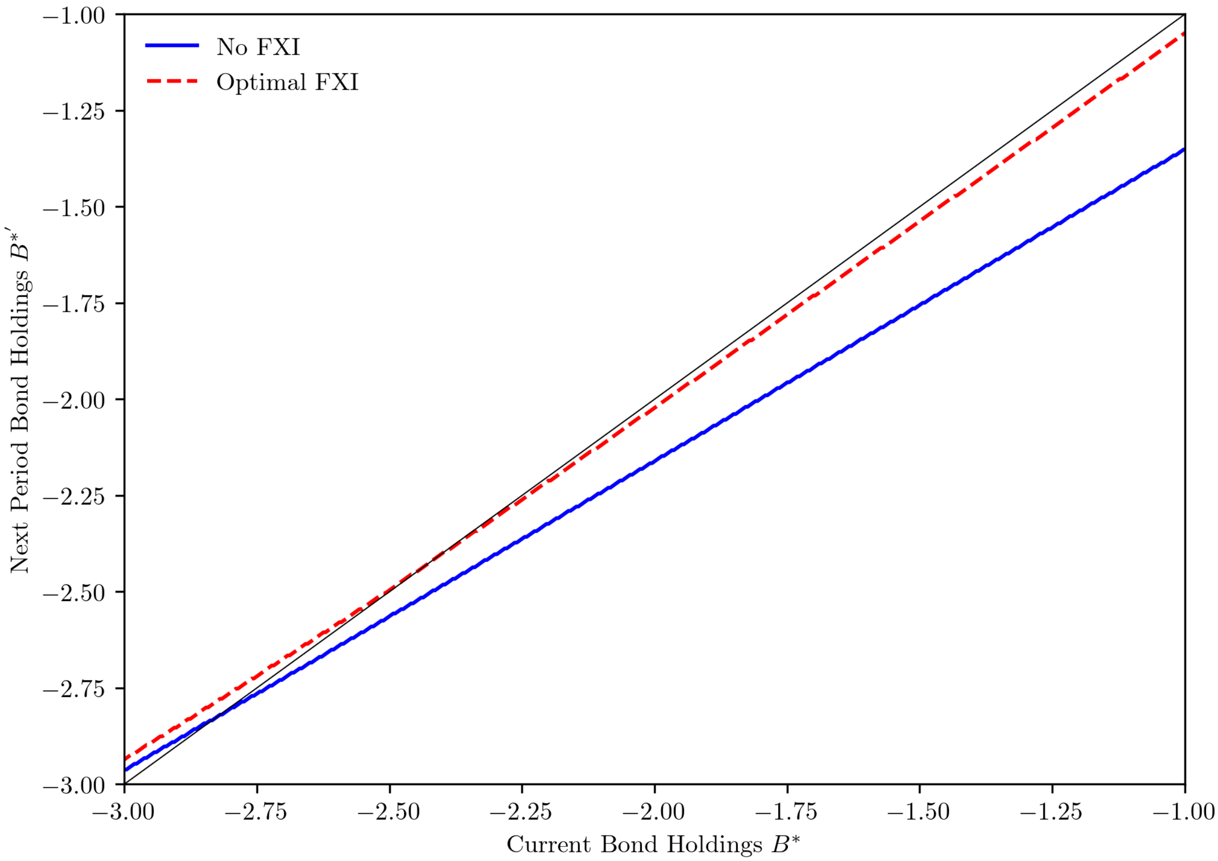

Figure 1: Policy function for net foreign assets B^*

Notes: The figure shows the policy function for bond holdings of the competitive equilibrium without FXI (blue solid line) and the constrained planner's solution (red dashed line) for the exogenous state $\{Y_T, Y_N, B_P^*\} = \{1.01, 1.02, -2.6\}$.

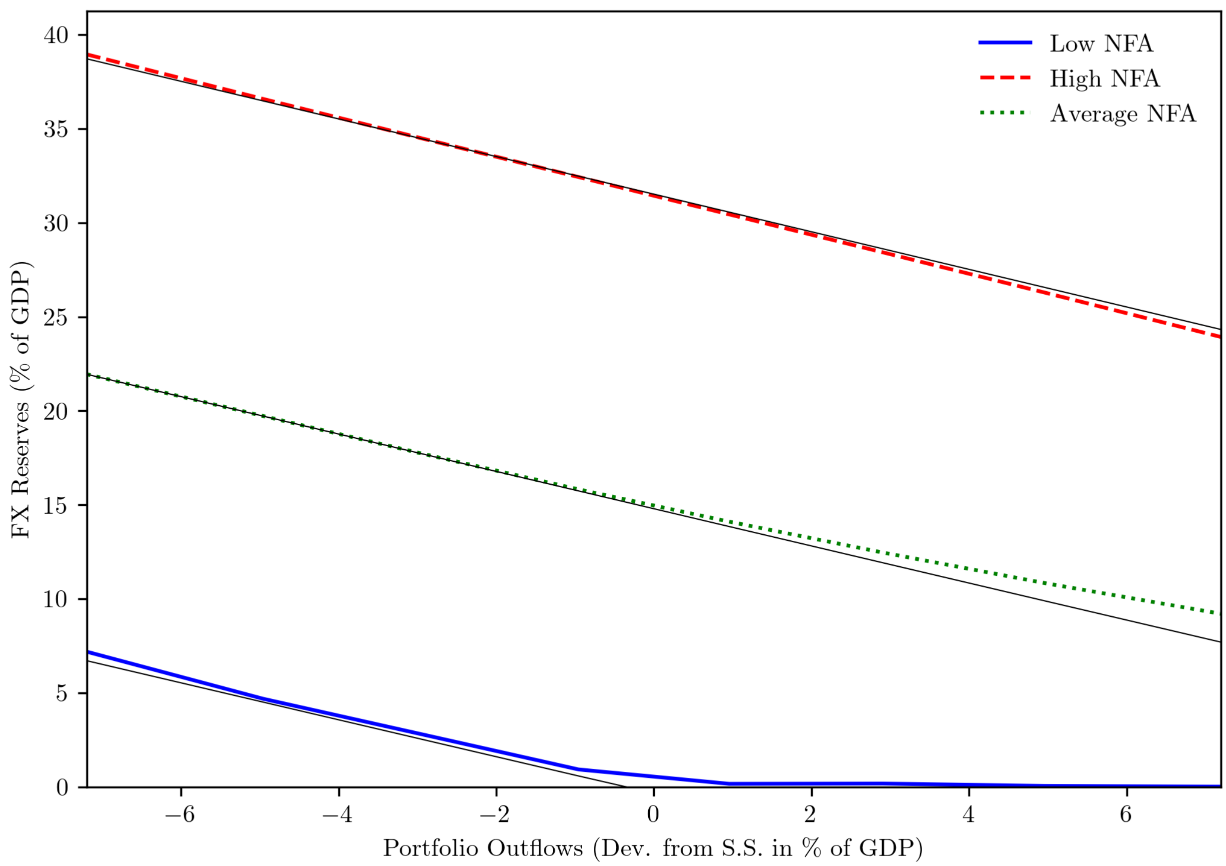

Figure 2: Policy function for FX reserves

Notes: The figure illustrates the policy functions for FX reserves of the constrained planner's solution conditional on relatively low current NFA (blue solid line) and relatively high NFA (red dashed line) as well as an average (green dotted line). The black solid lines depict the corresponding policy functions under the first-best. Low NFA is defined as the state $\{Y_T, Y_N, B^*\} = \{1.01, 1.02, -3.18\}$ and high NFA is defined as $\{Y_T, Y_N, B^*\} = \{1.01, 1.02, 0.16\}$. In terms of annual GDP, low and high NFA are -32% and 2%, respectively. The average is computed over all NFA positions B^*, conditional on $\{Y_T, Y_N\} = \{1.01, 1.02\}$.

the first-best benchmark, depending on the state. However, the average policy is shaped by a precautionary motive: conditional on the NFA position, the extent to which FXI offsets contemporaneous portfolio outflows declines with the size of the outflow shock.

4.3 Ergodic Implications

We next compare the ergodic distributions of key macroeconomic variables in an economy without FXI, to those corresponding to optimal, time-consistent FX interventions. To this end, we first generate a long sequence (2,000,000 periods) of the exogenous variables $\{Y_{T,t}, Y_{N,t}, B^*_{P,t}\}_{t=1}^{2\times10^6}$ based on the estimated Markov process described in Section 4.1. Using this exogenous sequence and the policy functions, we then compute equilibrium variables for the economy without FXI and with optimally conducted FXI.

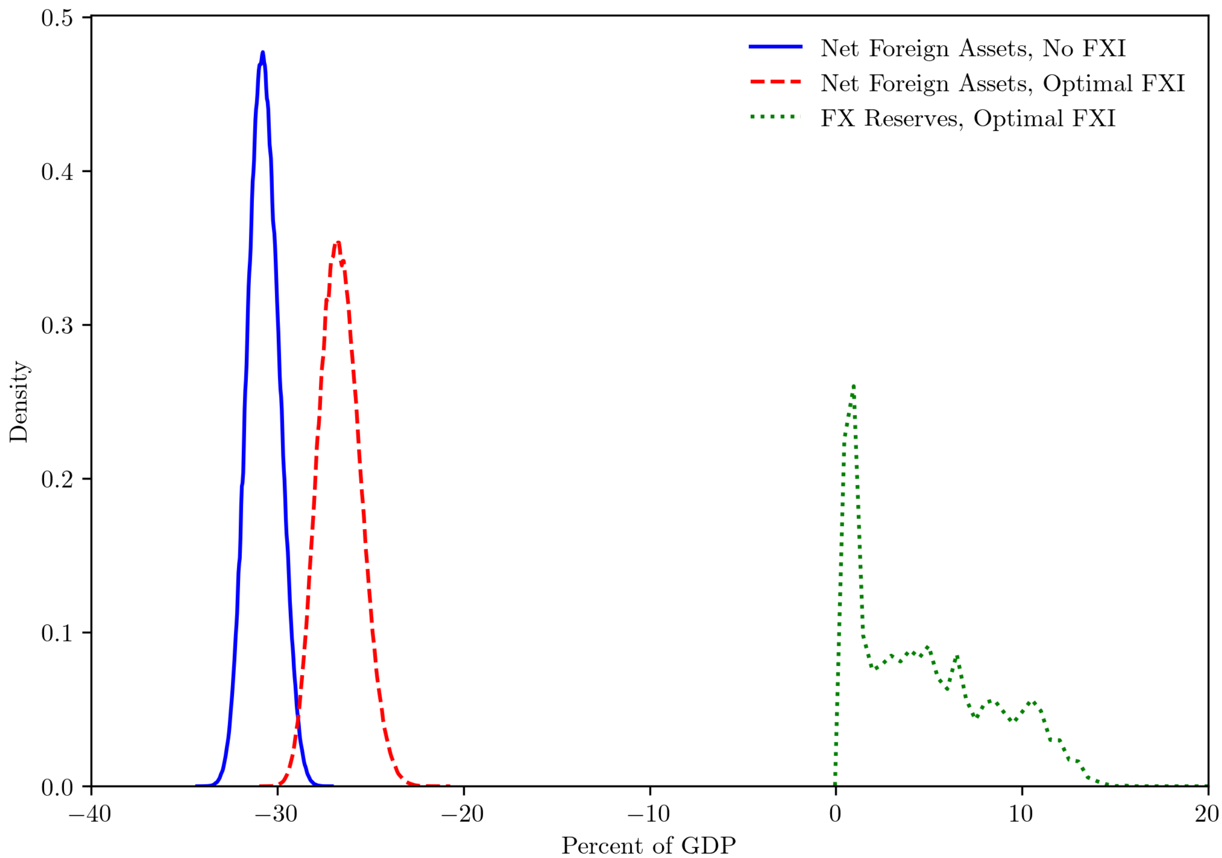

Figure 3: Ergodic distribution of net foreign assets and FX reserves

Notes: The figure shows the densities for the net foreign asset positions and FX reserves obtained by simulating the model for 2×10^6 quarters.

Figure 3 shows the unconditional distributions of the net foreign asset position under these two alternative policies, as well as of FX reserves under the optimal FXI policy. Strikingly, the optimal use of FXI significantly affects the steady state of the economy: savings are higher on average, resulting in a higher net foreign asset position. This occurs because active use of FXI implies a positive average level of FX reserves, which means that the demand for foreign currency increases. To accommodate it, bond market equilibrium (4) requires either lower borrowing by households (higher NFA B_t^*) or increased intermediation by financiers (higher $-B_{F,t}^*$). In our model, both occur in parallel. In particular, net foreign assets increase markedly, but not one-for-one with FX reserve holdings, indicating that FX market conditions also improve, allowing financiers to expand their balance sheets.

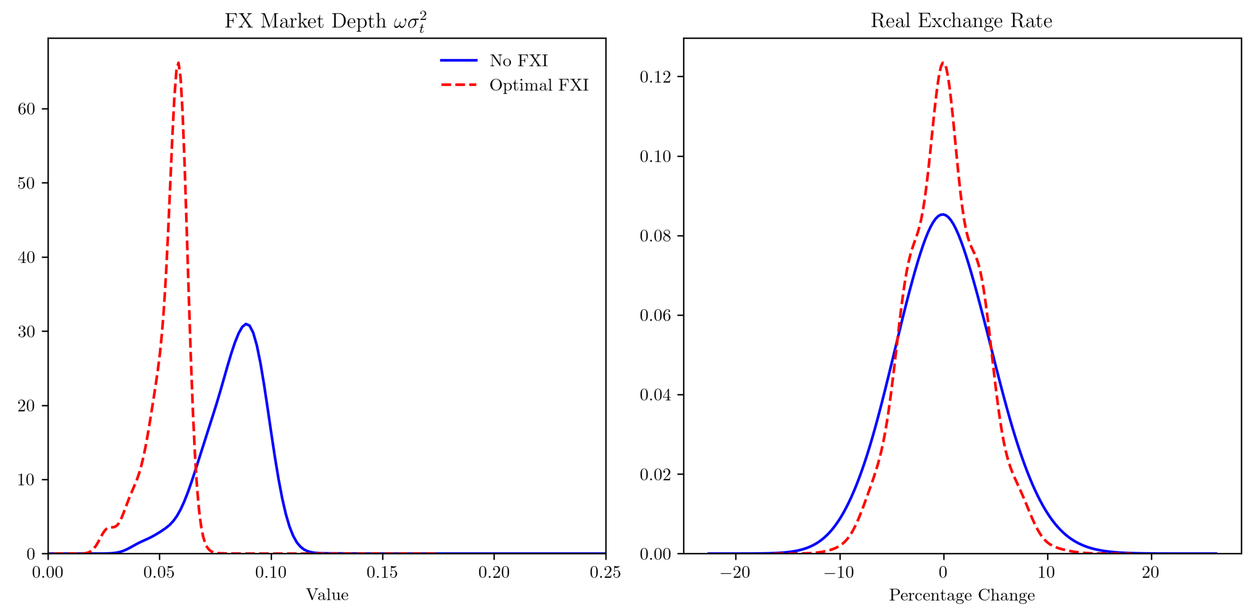

Figure 4: FX market conditions

Notes: The plot shows the densities for the FX market depth (left panel) and the real exchange rate (right panel) obtained from a simulation based on 2×10^6 quarters. The real exchange rate is defined as $\mathcal{Q}_t = \left[\alpha^\xi + (1-\alpha)^\xi \left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}$.

The drivers of the improvement in FX market conditions are illustrated in Figure 4, which shows the unconditional distribution of FX market depth $\omega\sigma_t^2$ (left panel), alongside the real exchange rate (right panel) for the two FXI regimes considered. Optimal FXI policy has a clearly stabilizing effect on the exchange rate. In essence, by counteracting *portfolio* flows, the central bank reduces the non-fundamental and inefficient part of exchange rate dynamics. However, the planner does allow for efficient exchange rate fluctuations in response to endowment shocks, which explains why there is still sizable exchange rate variability even under optimal policy. The left panel shows that the reduction in exchange rate risk ends up significantly deepening the FX market. We can summarize these observations as follows:

Remark 2. Compared to a no-FXI equilibrium, the stochastic steady state under the optimal time-consistent FXI policy is characterized by:

(a) a precautionary level of FX reserves and thus a higher net foreign asset position,

(b) deeper FX markets as the central bank acts as an FX liquidity provider.

4.4 Dynamics

We now analyze the model's dynamics, focusing on how portfolio flows affect the economy, and how their impact depends on whether foreign exchange interventions are deployed or not. To characterize an episode of portfolio capital outflows (inflows), we use the stochastic simulations described above, gathering all non-overlapping, 17-period samples, in which the peak outflow (inflow) occurs in the middle period, and averaging relevant macroeconomic variables over all the extracted subsamples.

Figure 5 depicts a typical portfolio outflow episode, with the top left panel showing that, at its peak (corresponding to $t = 0$), the outflow amounts to about 8% of steady-state GDP. In the absence of FX interventions (solid blue lines), a fall in the demand for domestic currency tightens external financial conditions and weakens the exchange rate. A higher UIP premium discourages borrowing from abroad, resulting in a sharp contraction of consumption. Moreover, portfolio outflows make FX markets shallower as illustrated by the bottom left panel of Figure 5. To understand the intuition, recall that market depth is inversely related to the risk borne by financiers, which is captured by the conditional exchange rate volatility σ_t^2. Holding their foreign currency borrowing fixed, a depreciation of the domestic currency scales up the (expected) payoff on their domestic currency position. This proportional increase magnifies both the mean and the variance of percentage returns — just as leverage would — thereby raising the required compensation for bearing risk.[18]

If the central bank follows optimal time-consistent FXI policy (dashed red lines), the reaction of FX reserves is almost the mirror image of portfolio outflows, although the offset is not full. More specifically, at the peak of portfolio outflows, in period $t = 0$, the central bank's reserves are down by around 5% of GDP, indicating that policymakers choose not to run reserves down completely. Nevertheless, this policy helps stabilize the economy since it significantly reduces the exchange rate depreciation and mitigates the deterioration in external financing conditions. As a result, the fall in consumption is also greatly reduced. Besides stabilizing

[18] This point is closely related to Proposition 1(a).

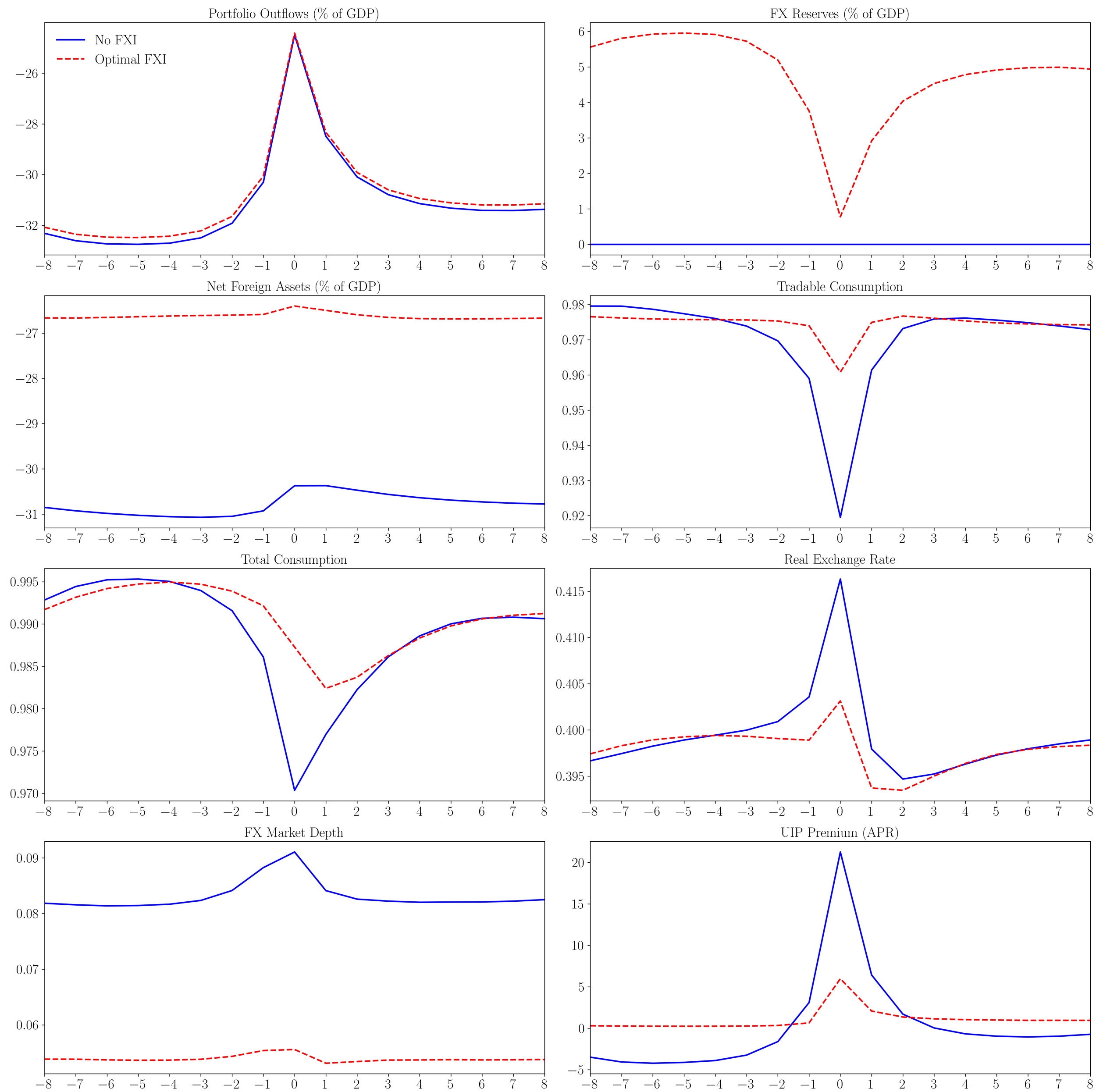

Figure 5: Portfolio outflow episode

Notes: The figure depicts the average paths of selected variables during an episode of portfolio outflows. One period corresponds to one quarter, and period 0 coincides with the peak outflow.

the exchange rate, the optimal FXI policy largely eliminates a spike in FX market depth. While this mainly reflects the fact that the intervention makes the currency stronger, it is also partially due to "keeping powder dry". Such policy conduct means that the monetary authority is better prepared to offset a possible future capital outflow, thus reducing the non-fundamental component of exchange rate risk.

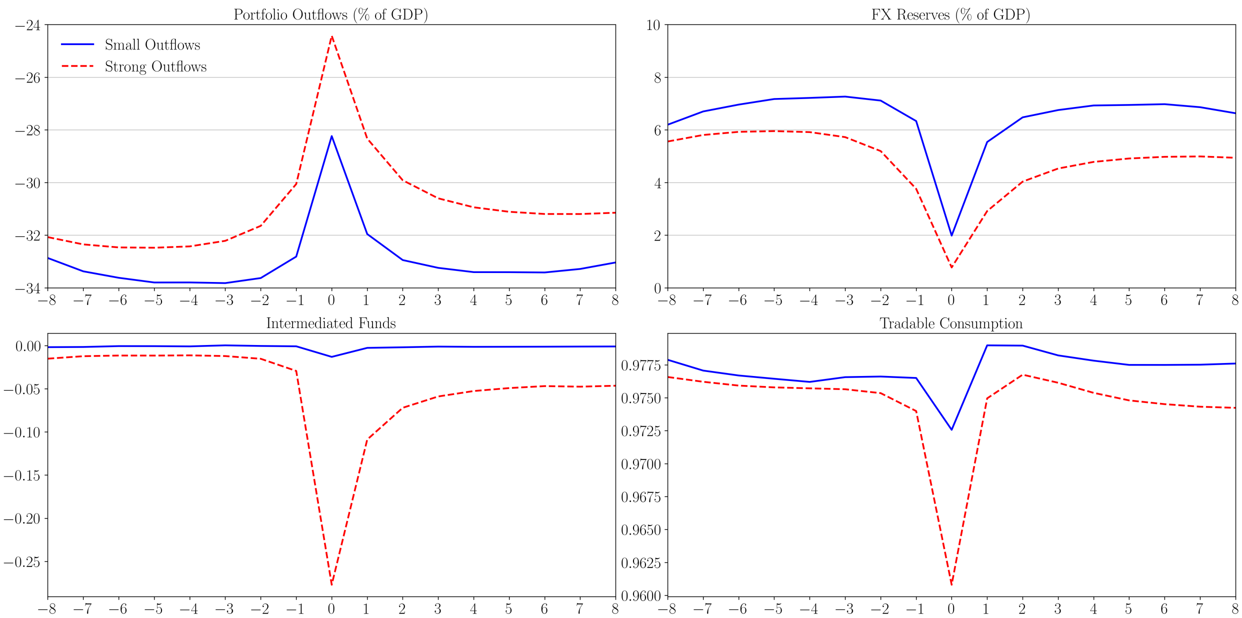

Figure 6: Optimal response to small and large portfolio outflows

Notes: The figure depicts the average paths of selected variables during episodes of small and large portfolio outflows. One period corresponds to one quarter and period 0 coincides with the peak outflow.

The extent to which optimal FXI policy offsets contemporaneous portfolio outflows depends on the size of (expected) portfolio outflows. While this offset is partial in Figure 5, it can also be more aggressive. We illustrate this point in Figure 6, where we distinguish between episodes of relatively small (blue solid lines) and large (red dashed lines) portfolio outflows. While the episodes of strong portfolio outflows are around four percentage points of GDP larger, the optimal FX intervention is similar, implying a considerably bigger offset. This asymmetry is also reflected in the amount of funds intermediated by financiers $B^*_{F,t}$. For strong portfolio outflows, which are only partially absorbed by the central bank's intervention, financiers increase their long exposure to domestic currency. This effect on their balance sheets almost vanishes in the case of small portfolio outflows. Because the central bank's FX sales nearly fully offset the portfolio outflows, the net demand for domestic currency is essentially unchanged. Notably, the documented asymmetry in the policy response does not simply reflect episodes in which the central bank runs out of reserves during periods of strong portfolio outflows. In fact, the pattern remains essentially unchanged when these episodes are excluded, highlighting the importance of the "keeping powder dry" motive.

While the previous figures were concerned with portfolio outflows, Figure 7 depicts the average model responses during portfolio inflow episodes. We see that the central bank offsets contemporaneous portfolio inflows almost one-for-one, thus following the first-best policy more closely. The underlying reason for that being the smaller relevance of the lower bound on reserves during inflow episodes, with the policy response mirroring the unconstrained case. In other words, optimal policy effectively stabilizes the economy during inflow episodes and mutes real exchange rate volatility.

Notably, and in contrast to portfolio outflows, inflows deepen FX markets. A corollary of this observation is that FX interventions are relatively more effective during episodes of portfolio outflows, as FX markets are generally shallower then. In other words, a central bank that follows the optimal FXI policy gets more bang for the buck while selling reserves than when purchasing them. The importance of these differences can be quantified by comparing Figures 5 and 7. There the central bank conducts FX sales (purchases) worth 4.8% (6.6%) of GDP between periods $t = -8$ and $t = 0$. However, the real exchange rate at the peak of outflows in period $t = 0$ is 3.5% more appreciated under optimal FXI compared to the decentralized equilibrium, while it is around 3.4% weaker in the case of inflows. Accordingly, in our simulations, FX sales have a 44% bigger impact on the real exchange rate than FX purchases, which we summarize in the following remark.

Remark 3. Under the optimal time-consistent FXI policy, FX sales are more effective than FX purchases as they are implemented when FX markets are relatively shallower.

Finally, we investigate the extent to which FX interventions should be used to respond to fundamental shocks, represented in our model by stochastic endowments. Figure 8 shows an average episode of endowment decreases. Starting with the case of no FXI (solid blue lines), the economy experiences a sharp contraction in consumption, which is only partially cushioned by increased borrowing from abroad. The real exchange rate appreciates because the fall in nontradable endowment (and thus nontradable consumption) dominates the decrease in tradable consumption.[19] Since the exogenous processes for endowments and portfolio outflows are slightly negatively correlated (see Section 4.1), portfolio capital flees the country.

If FXI is conducted optimally, it is focused on stabilizing the international risk-sharing wedge, which would otherwise reflect fluctuations in portfolio capital and the country's net foreign

[19] Figure C.1 in the Appendix summarizes episodes in which the tradable endowment falls while the nontradable endowment increases, in which case the real exchange rate depreciates.

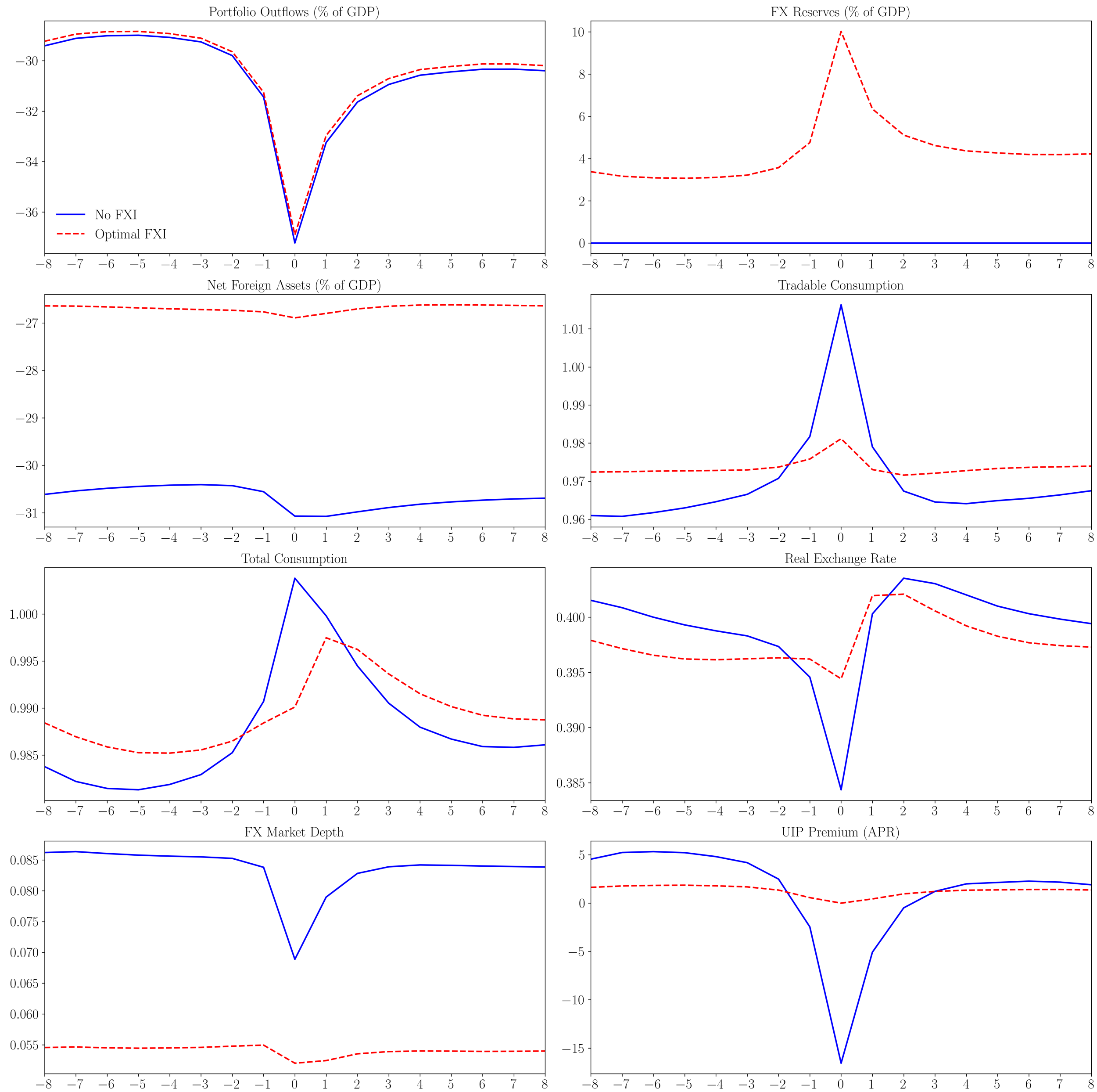

Figure 7: Portfolio inflow episode

Notes: The figure depicts the average paths of selected variables during an episode of portfolio inflows. One period corresponds to one quarter and period 0 coincides with the peak inflow.

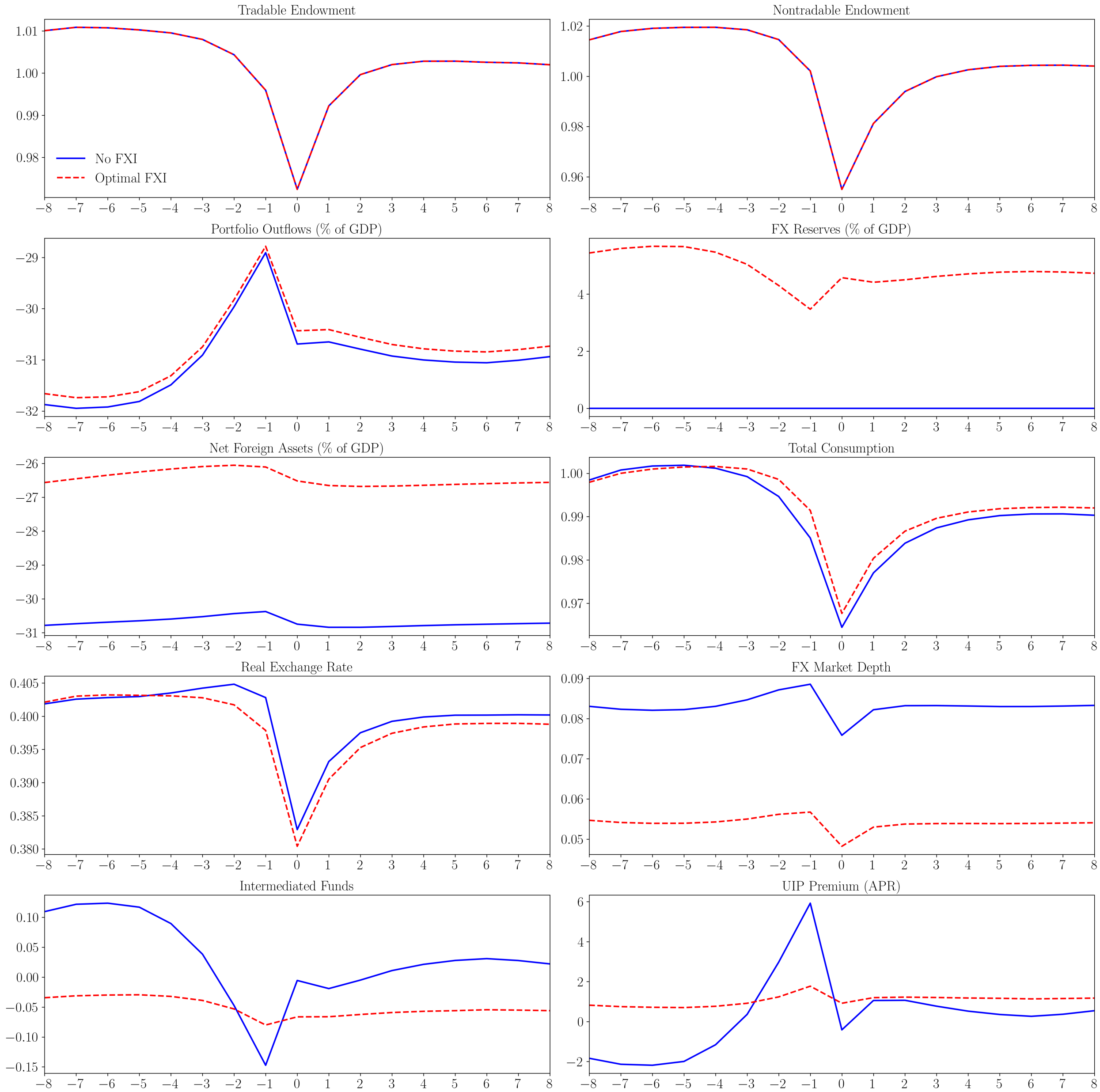

Figure 8: Episodes of negative endowment shocks

Notes: The figure depicts the average paths of selected variables during episodes of negative endowment shocks. One period corresponds to one quarter and period 0 coincides with the peak of the negative endowment shock.

asset position. However, the policy is not effective in limiting the fall in consumption that is driven by fundamental forces. Its effect on the exchange rate adjustment is also small as the latter facilitates efficient expenditure switching. We summarize these observations in the following remark.

Remark 4. In response to fundamental shocks, the optimal time-consistent FXI policy focuses on stabilizing the international risk-sharing wedge, otherwise allowing the exchange rate to float freely.

5 Welfare Implications

So far, we have seen how optimal FXI can efficiently stabilize the economy by reducing excessive exchange rate volatility arising from cross-border portfolio flows. In this section, we compute the welfare implications of this policy and compare them to those of alternative policy regimes.

As is standard in the literature, we express welfare gains in consumption equivalence units. Formally, let κ be the additional fraction of consumption that households in the benchmark economy b would have to receive to make them indifferent to living in an economy with alternative policy p. Given our assumptions on the utility functional, κ_p can be computed as

$$\kappa_p = \left(\frac{\tilde{V}_p \left(\overline{B}^* \right)}{\tilde{V}_b \left(\overline{B}^* \right)} \right)^{\frac{1}{1-\sigma}} - 1, \tag{16}$$

where $\tilde{V}_i$ denotes the household lifetime utility under policy regime $i \in \{b, p\}$, conditional on initial bond holdings that coincide with their steady state value in the benchmark economy $\overline{B}^*$, and averaged over the stationary distribution of the exogenous state process (more details can be found in Appendix C). Since the measure conditions on the benchmark economy's steady state, the welfare gain κ_p takes into account transitional dynamics, i.e., it is not simply based on an unconditional comparison of welfare in the two steady states.

We chose the version of the model calibrated to Malaysia – which accounts for BNM's observed FX interventions – as our benchmark. We compare it to several alternative FXI policy regimes. These include the optimal time-consistent FXI policy, as described in Section 3, as well as two alternatives in which the central bank commits to simple FXI rules, which respect the lower bound on reserves. The reason for considering simple rules is that the optimal intervention formula (12) is very complicated and includes expectations, which may make it difficult to implement.[20] Finally, to provide an upper bound on welfare gains, including those achievable under commitment, we also consider the first-best policy.[21] As regards the simple rules, the first one is inspired by the first-best policy, with the crucial distinction that the central bank is subject to the LBR

$$B_{M,t}^{*} = \max\left(B_t^{*} - B_{P,t}^{*}, 0\right). \tag{17}$$

Note that this policy, which we will refer to as the UIP premium rule, eliminates the international risk-sharing wedge whenever feasible. The second simple rule, which we refer to as the portfolio flow rule, offsets only the exogenous component of portfolio flows

$$B_{M,t}^{*} = \max\left(-B_{P,t}^{*}, 0\right). \tag{18}$$

These two FXI rules are interesting for several reasons. First, as outlined in Section 3, the optimal time-consistent policy resembles the UIP rule, but deviates from it by accounting for the possibility that the LBR might bind in the future. A quantitative comparison of these two policy regimes helps us evaluate the significance of these forward-looking considerations. Second, comparing the UIP rule to the portfolio flow rule allows us to assess the welfare implications of stabilizing two components of the international risk-sharing wedge: the endogenous one associated with the net foreign asset position B_t^*, and the exogenous one represented by the portfolio capital position $B_{P,t}^*$.

Table 2 reports the welfare gains for the four policy regimes described above. Moving to the optimal time-consistent FXI policy improves welfare by 0.25% of consumption. This comes quite close to the maximum achievable gain of 0.29% represented by the first-best. The rule-based policy regimes also enhance welfare, albeit to a lesser extent, with the UIP and portfolio flow rules yielding welfare gains of 0.15% and 0.05%, respectively. Overall, the optimal use of FXI significantly outperforms both of the considered rules, and is only slightly worse than the first-best. This highlights the quantitative importance of the forward-looking element in the optimal time-consistent FXI policy, and it suggests that the scope for further gains achievable through commitment is limited. However, and as we shall explain, the latter

[20] That feature of the optimal time-consistent FXI policy in our model is shared with the optimal tax on debt formula that decentralizes the planner's allocation in models with collateral constraints, see, e.g., Bianchi & Mendoza (2018).

[21] To solve for the first-best policy, we posit that FX interventions are characterized by equation (9), which effectively assumes away the LBR. The associated equilibrium is stationary since households are impatient ($\beta R^* < 1$) and there is a lower bound on the net foreign asset position, which jointly guarantee that the transversality condition is satisfied. More details on the recursive representation of the first-best are provided in Appendix C.

part of this conclusion crucially depends on the initial level of FX reserves.

Table 2: Welfare Implications of Alternative FXI Policies

	Portfolio Rule	UIP Premium Rule	Time-Consistent Optimal FXI	First Best
Welfare Gain	0.045	0.150	0.248	0.288

Notes: The welfare gain is computed according to formula (16) and expressed in percentages. The UIP and portfolio flow rules are defined by equations (17) and (18), respectively.

To clarify the differences between the alternative FXI policies, it is useful to examine their ergodic implications. Table 3 presents a selection of first and second moments obtained from long stochastic simulations.[22] One point worth emphasizing is that the average stock of FX reserves varies significantly across the different policy regimes. This further translates into differences in the net foreign asset position, since FX reserves, by increasing the net demand for foreign currency, partially crowd out borrowing by domestic households.

We have already shown, in Section 4.3, that optimal time-consistent FXI policy results in moderately positive levels of FX reserves. Due to the precautionary motive, their average level is higher than under the closely related UIP rule, but the difference is relatively small. However, this small difference, coupled with intervention patterns designed to "keep the powder dry", is enough to drastically reduce the frequency of episodes in which FX reserves are depleted. FX markets are also deeper and the volatility of the risk-sharing wedge is substantially reduced if FXI is conducted optimally rather than according to the UIP rule.

Table 3: Unconditional Moments under Alternative FXI Regimes

	No FXI		Optimal FXI		UIP Rule		Portfolio Rule		First Best	
Variable Name	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Consumption	0.988	0.025	0.990	0.020	0.990	0.019	1.000	0.022	0.987	0.023
Net Foreign Assets	-0.307	0.008	-0.266	0.011	-0.271	0.017	-0.006	0.008	-0.337	0.007
FX Reserves	0.000	0.000	0.046	0.038	0.041	0.038	0.301	0.043	-0.029	0.044
Real Exchange Rate	0.399	0.014	0.398	0.012	0.398	0.013	0.390	0.011	0.400	0.011
FX Market Depth	0.083	0.013	0.054	0.008	0.058	0.009	0.047	0.008	0.042	0.010
UIP Premium	0.007	0.127	0.012	0.028	0.012	0.032	0.012	0.012	0.000	0.000
Reserve Depletion Freq.	–		0.020		0.168		0.000		–	

Notes: The moments are obtained by simulating each FXI regime over 2×10^6 quarters. NFA and FX reserves are expressed as a fraction of annual GDP. The risk sharing wedge is annualized.

[22] Figure C.2 in Appendix C compares how the economy behaves under the different policy regimes during portfolio outflow episodes.

If the FXI rule only responds to the portfolio component of the UIP premium, FX reserves are very high. This simply reflects the fact that, according to our calibration, there is positive appetite for domestic currency on average ($B^*_{P,t}$ is typically negative), while the economy's net foreign assets position (i.e., the part of the UIP premium that the rule ignores) is negative, as domestic households are relatively impatient ($\beta R^* < 1$). The high level of reserves implies that the risk of depleting them is virtually nil. As a consequence, exchange rate volatility is much lower and FX markets are deeper compared to the optimal policy. This ultimately translates into smaller fluctuations in the international risk sharing wedge, even though the rule does not respond to its endogenous component associated with the NFA. However, these stabilization gains come at the cost of low international borrowing (the steady state NFA position is nearly balanced), which is the main reason why this policy is associated with relatively modest welfare gains.

In the first-best, the central bank does not need to accumulate precautionary FX reserves since its use of FXI is unconstrained. In fact, it typically borrows from abroad in order to accommodate the borrowing needs of domestic households, so that the country's NFA position is lower than in the other FXI regimes. Notably, the first-best policy perfectly eliminates the risk-sharing wedge, but it does not stabilize the exchange rate much beyond what the portfolio rule achieves, ultimately reflecting the expenditure switching motive in response to fundamental shocks. As a consequence, FX markets are deeper than under any of the other policies considered, but not perfectly deep.

One complication in interpreting the welfare gains reported in Table 2 is that the various policy regimes imply different steady states. Recall that the initial NFA position used to compute welfare gains, as defined in (16), corresponds to the observed economy. Malaysia's average stock of FX reserves equal to 30% of annual GDP translates into an NFA position of about 2% of annual GDP. In the context of our model, this level of reserves is excessive and the resulting international debt too small. Consequently, a gradual decumulation of reserves during the transition period contributes positively to the welfare gains reported in Table 2. Figure 9 illustrates this point by comparing the value functions under different policy regimes as a function of the initial NFA position. Clearly, starting from lower level of assets means that the optimal time-consistent FXI policy deviates more from the first-best and the degree to which it overperforms the UIP premium rule shrinks, turning into a small disadvantage for a sufficiently high level of initial debt.

To isolate the welfare effects beyond those arising from transitional reserve decumulation, we conduct a comparison where the considered policy regimes share a common steady-state NFA position. Specifically, we redefine the benchmark economy to be the optimal time-

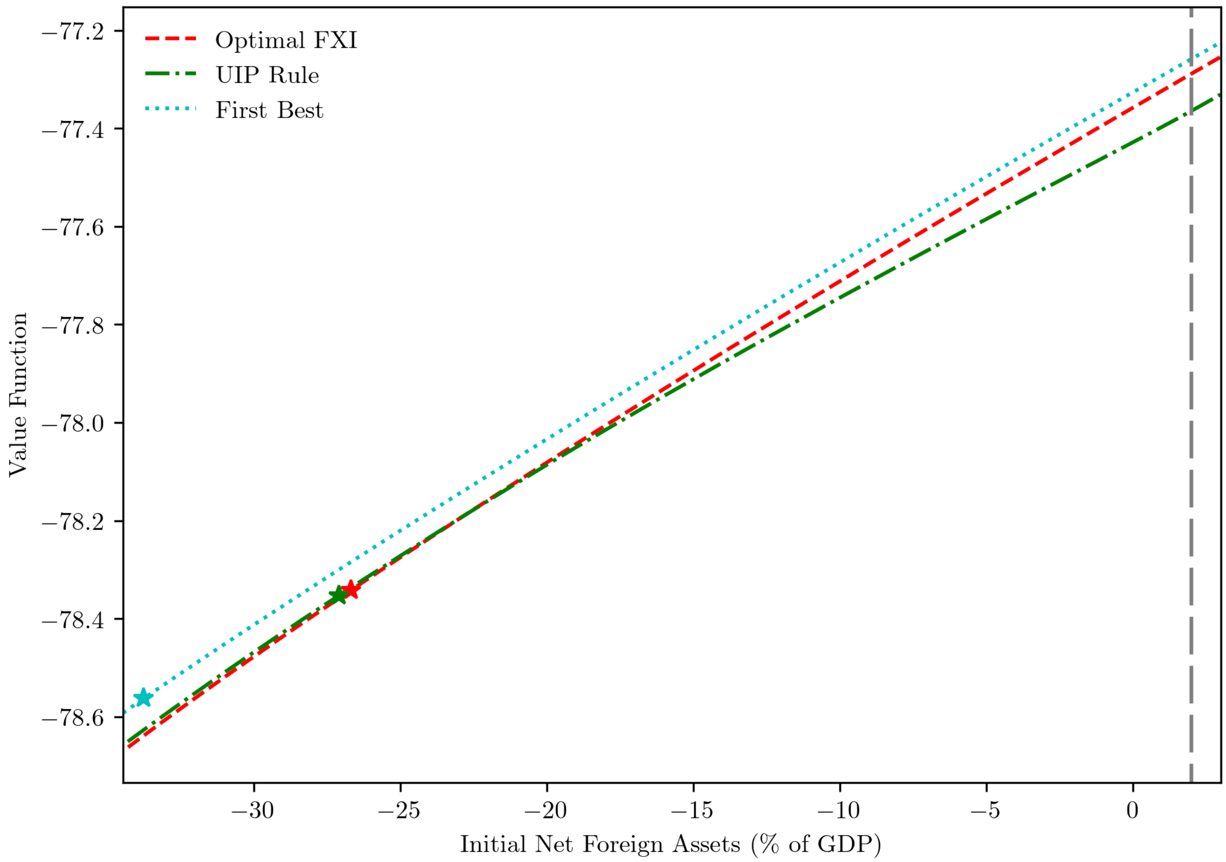

Figure 9: Value Functions of Alternative FXI Policies

Notes: The figure depicts value functions $\tilde{V}_i$ of alternative policy regimes. The vertical dashed line corresponds to the net foreign asset position (or bond holdings) in the stochastic steady state of the calibrated economy. The stars indicate the net foreign asset positions in the stochastic steady state of the respective alternative FXI policies.

consistent policy and adjust the stock of FX reserves in each alternative policy regime to match the steady-state NFA position under this benchmark. Accordingly, the UIP premium rule defined in (17) is modified as follows

$$B^*_{M,t} = \max\left(\gamma_{UIP} + B^*_t - B_{P,t^*}, 0\right), \tag{19}$$

where γ_{UIP} is a constant term that achieves the desired average NFA level. We also consider a policy aimed at smoothing the real exchange rate, which we will refer to as the RER rule. This rule may seem appealing to policymakers as it relies on readily observable variables and we define it as

$$B^*_{M,t} = \max\left(\gamma_{RER} - \phi\left(\frac{\mathcal{Q}_t - \overline{\mathcal{Q}}}{\overline{\mathcal{Q}}}\right), 0\right), \tag{20}$$

where $\phi \geq 0$ controls how aggressively the central bank leans against the fluctuations in the real exchange rate, the steady state level of which is denoted by $\overline{\mathcal{Q}}$. We calibrate ϕ such that it maximizes welfare and, as before, γ_{RER} is set to ensure that the steady-state NFA matches that under the optimal time-consistent policy. Finally, we consider a "no FXI" regime, which corresponds to a special case of the rule above where $\phi = 0$.

Table 4: Welfare Costs of FXI Rules relative to optimal time-consistent policy

	No FXI	RER Rule	UIP Rule
Welfare Cost	0.029	0.011	-0.003

Notes: The welfare cost is expressed in percentages and computed as $-\kappa_p$ in formula (16), where the benchmark economy b is the optimal time-consistent policy. All considered FXI rules have identical NFA positions in the steady state as described in the text.

Table 4 presents the welfare costs of the three considered FXI rules relative to the optimal time-consistent policy. Notably, now that the steady-state NFA positions are identical across all policies, the welfare differences are substantially smaller than those reported in Table 2. This underscores the significant role played by the average level of FX reserves in shaping the welfare outcomes. The optimal time-consistent policy still improves welfare compared to a regime in which FX reserves are kept constant, but the gain expressed in consumption equivalence units is approximately 3 basis points. Relative to the (optimized) RER rule, the gain is about three times smaller but still positive, reflecting the importance of distinguishing between inefficient volatility of the exchange rate caused by portfolio flow shocks and its efficient adjustment to endowment shocks.

Interestingly, despite being purely static and hence missing forward-looking motives of the

FXI conduct, the UIP rule yields welfare that is very similar to that under the optimal time-consistent policy, even outperforming it by a small margin. This outcome reflects the importance of time inconsistency in the model. As explained in Section 3, the optimal time-consistent policy does not coincide with the optimal policy under commitment, which would entail a "forward guidance" mechanism, nor is it guaranteed to outperform some simple rules. While the value of commitment is small when the economy is far away from the LBR, it becomes sufficiently big for the simple UIP rule to prevail over the optimal time-consistent policy when the average level of FX reserves is only around 5% of GDP, as assumed in the comparison presented in Table 4.

The following remark concludes our analysis of the welfare implications of alternative FXI regimes.

Remark 5. If the initial stock of FX reserves is high, adopting the optimal time-consistent FXI policy delivers robust welfare gains over simple rule-based alternatives, with performance close to the first-best. However, if FX reserves are initially small, the optimal time-consistent policy achieves welfare that is very similar to a simple rule targeting ex-ante UIP deviations.

6 Conclusions

This paper analyzes the optimal use of foreign exchange interventions in a small open economy with endogenous FX market depth and a lower bound on FX reserves. We find that optimal time-consistent FXI can effectively reduce exchange rate volatility caused by portfolio flow shocks, thereby shrinking deviations from uncovered interest parity and improving FX market depth. The optimal policy response hinges on the expected path of portfolio flows and the existing net foreign asset position. When the economy has a relatively strong NFA position and/or outflows are moderate and expected to ease, it can be optimal to hold FX reserves below their first-best level. In contrast, when outflows are larger and likely to persist, and the NFA position is weak, the precautionary motive dominates, making it optimal to maintain higher reserve buffers.

Our quantitative analysis further reveals that under optimal policy, the effectiveness of FXI is state-dependent. Specifically, FX purchases tend to have a lower impact on the exchange rate as they occur during periods of capital inflows, when FX markets are deeper, while FX sales are relatively more effective as they take place amid portfolio outflows, when markets tend to be shallower. We also find that the optimal time-consistent FXI policy is associated with substantial welfare gains, at least if the economy starts out with a sufficiently high level of FX reserves. Committing to an FXI rule that aims to smooth UIP deviations also improves welfare, albeit to a lesser extent. However, the value of commitment stemming from the lower bound on reserves becomes more pronounced when reserves are relatively low. As a result, the optimal time-consistent FXI policy may no longer outperform the simple UIP rule in such circumstances.

Overall, our analysis provides a tractable quantitative framework to analyze the use of FXI. It highlights the importance of a precautionary motive behind reserve accumulation, as well as state dependency in the conduct of FXI policy. Future research could build on our results, e.g., by incorporating additional financial and nominal rigidities or endogenizing the effective lower bound on FX reserves, to see how such changes affect the optimal deployment of FXI.

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Appendix

A Proofs

Proof of Proposition 1 We first note that the resource constraint implies $\partial B_t^*/\partial B_{P,t}^* = -\partial C_{T,t}/\partial B_{P,t}^*$. We now proceed by proving the two parts of the proposition. Starting with point (a), the expression for the conditional exchange rate volatility can be rewritten as

$$\sigma_t^2 = R_t^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \left(\frac{1}{\mathbb{E}_t\left[\Theta_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]}\right)^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = \frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right),$$

where we have used the Euler equation in the second line. The partial derivative with respect to consumption in period t is therefore given by

$$\frac{\partial\sigma_t^2}{\partial C_{T,t}} = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right) = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\sigma_t^2.$$

Taking the partial derivative with respect to $B_{P,t}^*$ and using the previous equations we obtain the result in question

$$\frac{\partial\sigma_t^2}{\partial B_{P,t}^*} = \frac{\partial\sigma_t^2}{\partial C_{T,t}}\frac{\partial C_{T,t}}{\partial B_{P,t}^*} = 2\overbrace{\left(\frac{u_{11,t}}{u_{1,t}}\right)}^{<0}\sigma_t^2\overbrace{\left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)}^{<0} > 0.$$

Turning to property (b), suppose by contradiction that $\frac{\partial\mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*} \geq 0$, which would then imply

$$\begin{aligned}\frac{\partial\mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B_{P,t}^*} &= \frac{\partial\mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial C_{T,t}}\frac{\partial C_{T,t}}{\partial B_{P,t}^*} = \left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\Theta_{t+1}\right]\frac{\partial C_{T,t}}{\partial B_{P,t}^*} \geq 0 \\ &\Rightarrow \frac{\partial C_{T,t}}{\partial B_{P,t}^*} \geq 0 \Rightarrow \frac{\partial B_t^*}{\partial B_{P,t}^*} \leq 0.\end{aligned}$$

However, this would violate our assumption that $\frac{\partial B_t^*}{\partial B_{P,t}^*} > 0$, and thus it must be the case that $\frac{\partial\mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*} < 0$. ■

Derivation of $\partial B_t^*/\partial B_{P,t}^*$ Taking the partial derivative of the IRS condition with respect to $B_{P,t}^{*}$ yields

$$\omega \frac{\partial \sigma_t^2}{\partial B_{P,t}^*}\left(B_t^* - B_{P,t}^*\right) + \omega \sigma_t^2 \left(\frac{\partial B_t^*}{\partial B_{P,t}^*} - 1\right) = R^* \frac{\partial \mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B_{P,t}^*}.$$

Inserting the expressions for $\frac{\partial \sigma_t^2}{\partial B_{P,t}^*}$ and $\frac{\partial \mathbb{E}_t[\Theta_{t+1}]}{\partial B_{P,t}^*}$ from the proof of Proposition 1 we then obtain

$$2\omega \left(\frac{u_{11,t}}{u_{1,t}}\right) \sigma_t^2 \left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)\left(B_t^* - B_{P,t}^*\right) + \omega \sigma_t^2 \left(\frac{\partial B_t^*}{\partial B_{P,t}^*} - 1\right) = -R^* \left(-\frac{u_{11,t}}{u_{1,t}}\right) \mathbb{E}_t\left[\Theta_{t+1}\right] \frac{\partial B_t^*}{\partial B_{P,t}^*}.$$

Further plugging in the IRS condition $R^* \mathbb{E}_t\left[\Theta_{t+1}\right] = 1 + \omega \sigma_t^2 \left(B_t^* - B_{P,t}^*\right)$ on the RHS we get

$$2\omega \left(\frac{u_{11,t}}{u_{1,t}}\right) \sigma_t^2 \left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)\left(B_t^* - B_{P,t}^*\right) + \omega \sigma_t^2 \left(\frac{\partial B_t^*}{\partial B_{P,t}^*} - 1\right) = -\left(-\frac{u_{11,t}}{u_{1,t}}\right) \frac{\partial B_t^*}{\partial B_{P,t}^*} \left(1 + \omega \sigma_t^2 \left(B_t^* - B_{P,t}^*\right)\right).$$

After rearranging, we have

$$\omega \sigma_t^2 \frac{\partial B_t^*}{\partial B_{P,t}^*}\left[\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_t^* - B_{P,t}^*\right) + \frac{1}{\omega \sigma_t^2}\right) + 1\right] = \omega \sigma_t^2.$$

Finally, rewriting the equation as an expression for $\frac{\partial B_t^*}{\partial B_{P,t}^*}$

$$\frac{\partial B_t^*}{\partial B_{P,t}^*} = \frac{1}{1 - \left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_{P,t}^* - B_t^*\right) - \frac{1}{\omega \sigma_t^2}\right)},$$

implies that the sufficient condition for $\frac{\partial B_t^*}{\partial B_{P,t}^*} > 0$ is given by

$$\frac{\partial B_t^*}{\partial B_{P,t}^*} > 0 \Leftrightarrow \left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3\left(B_{P,t}^* - B_t^*\right) - \frac{1}{\omega \sigma_t^2}\right) < 1.$$

Proof of Theorem 1 The Lagrangian of the second-best can be written as

$$\begin{aligned} \mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \Bigg[& u\left(B_{t+s-1}^* R^* - B_{t+s}^* + Y_{T,t+s}, Y_{N,t+s}\right) \\ & - \lambda_{t+s}\left(B_{P,t+s}^* + \frac{1}{\omega \sigma_{t+s}^2}\left(R^* \mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right] - 1\right) - B_{t+s}^*\right)\Bigg]. \end{aligned}$$

The first-order condition with respect to B_t^* yields (11) in the main text

$$
\begin{aligned}
&u_{1,t}-\lambda_t\left(1-\frac{\partial\sigma_t^2}{\partial B_t^*}\frac{-1}{\omega(\sigma_t^2)^2}\left(R^*\mathbb{E}_t\left[\Theta_{t+1}\right]-1\right)-\frac{R^*}{\omega\sigma_t^2}\mathbb{E}_t\left[\frac{\partial\Theta_{t+1}}{\partial B_t^*}\right]\right)\\
=&\beta R^*\mathbb{E}_t\left[u_{1,t+1}\right]-\beta\mathbb{E}_t\left[\lambda_{t+1}\frac{\partial\sigma_{t+1}^2}{\partial B_t^*}\frac{-1}{\omega\left(\sigma_{t+1}^2\right)^2}\left(R^*\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]-1\right)\right]-\beta\mathbb{E}_t\left[\lambda_{t+1}\frac{R^*}{\omega\sigma_{t+1}^2}\mathbb{E}_{t+1}\left[\frac{\partial\Theta_{t+2}}{\partial B_t^*}\right]\right].
\end{aligned}
$$

Note that σ_{t+1}^2 can be written as

$$
\begin{aligned}
\sigma_{t+1}^2=R_{t+1}^2 var_{t+1}\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right)=&\left(\frac{1}{\mathbb{E}_{t+1}\left[\Theta_{t+2}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right]}\right)^2 var_{t+1}\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_{t+2}}\right)\\
&=\frac{\left(u_{1,t+1}\right)^2}{\left(\mathbb{E}_{t+1}\left[\beta\frac{u_{1,t+2}}{\mathcal{E}_{t+2}}\right]\right)^2}var_{t+1}\left(\frac{1}{\mathcal{E}_{t+2}}\right),
\end{aligned}
$$

which yields the following partial derivative with respect to B_t^*

$$
\frac{\partial\sigma_{t+1}^2}{\partial B_t^*}=2R^*\left(\frac{u_{11,t+1}}{u_{1,t+1}}\right)\sigma_{t+1}^2<0. \tag{A.1}
$$

Furthermore, the partial derivative of the stochastic discount factor Θ_{t+2} with respect to B_t^* is given by

$$
\frac{\partial\Theta_{t+2}}{\partial B_t^*}=R^*\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\Theta_{t+2}>0. \tag{A.2}
$$

Next, inserting (A.1) and (A.2) into the first-order condition derived above, we obtain

$$
\begin{aligned}
u_{1,t}=&\beta R^*\mathbb{E}_t\left[u_{1,t+1}\right]\\
&-\beta R^*\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)2\left(B_{t+1}^*-B_{P,t+1}^*\right)\right]-\beta R^*\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\frac{R^*}{\omega\sigma_{t+1}^2}\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]\right].
\end{aligned}
$$

Plugging the period $t+1$ IRS condition $\frac{R^*}{\omega\sigma_{t+1}^2}\mathbb{E}_{t+1}\left[\Theta_{t+2}\right]=\frac{1}{\omega\sigma_{t+1}^2}+\left(B_{t+1}^*-B_{P,t+1}^*\right)$ into the third term on the RHS yields

$$
\begin{aligned}
u_{1,t}=\beta R^*\mathbb{E}_t\Bigg[&u_{1,t+1}-\lambda_{t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)2\left(B_{t+1}^*-B_{P,t+1}^*\right)\\
&-\lambda_{t+1}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(\frac{1}{\omega\sigma_{t+1}^2}+\left(B_{t+1}^*-B_{P,t+1}^*\right)\right)\Bigg].
\end{aligned}
$$

Rearranging and simplifying then implies

$$1 - R^* \mathbb{E}_t \left[\Theta_{t+1}\right] = \frac{\beta R^*}{u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right].$$

Finally, plugging in the period t IRS condition $R^* \mathbb{E}_t \left[\Theta_{t+1}\right] = 1 + \omega \sigma^2_t \left(B^*_t - B^*_{P,t} - B^*_{M,t} \right)$ and rearranging gives

$$B^*_{M,t} = B^*_t - B^*_{P,t} + \frac{\beta R^*}{\omega \sigma^2_t u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right],$$

which completes the proof. ■

Proof of Proposition 2 From Theorem 1, the optimal level of reserves is higher relative to the first-best if and only if

$$\begin{aligned}
& B^*_{M,t} > B^*_t - B^*_{P,t} \\
\Leftrightarrow & \frac{\beta R^*}{\omega \sigma^2_t u_{1,t}} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(-3 B^*_{F,t+1} - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(\omega \sigma^2_{t+1} B^*_{F,t+1} + \frac{1}{3} \right) \right] < 0,
\end{aligned}$$

where we have made use of $B^*_{F,t} = B^*_{t+1} - B^*_{P,t+1}$. Next, applying the IRS condition $R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right] - 1 = \omega \sigma^2_{t+1} B^*_{F,t+1}$ we obtain

$$\begin{aligned}
& \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right] - \frac{2}{3} \right) \right] < 0 \\
\Leftrightarrow \; & \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right] \mathbb{E}_t \Big[R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right] \Big] + cov \left(\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right] \right) \\
& \qquad < \frac{2}{3} \mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right].
\end{aligned}$$

Finally, dividing through by $\mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]$ and using the law of iterated expectations we arrive at

$$R^* \mathbb{E}_t \left[\Theta_{t+2}\right] < \frac{2}{3} - \frac{cov \left(\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^* \mathbb{E}_{t+1} \left[\Theta_{t+2}\right] \right)}{\mathbb{E}_t \left[\lambda_{t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]},$$

which completes the proof. ■

Derivation of the Absolute Risk Aversion The marginal utility with respect to tradables is given by

$$u_{1,t} = \alpha C_t^{-\sigma} \left(\frac{C_t}{C_{T,t}} \right)^{\frac{1}{\xi}} = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}}.$$

Taking the derivative with respect to tradables yields

$$u_{11,t} = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}} \left(\frac{1-\sigma\xi}{\xi} C_t^{-1} \frac{\partial C_t}{\partial C_{T,t}} - \frac{1}{\xi} C_{T,t}^{-1} \right) = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}} \left(\frac{1-\sigma\xi}{\xi} C_t^{-1} \alpha \left(\frac{C_t}{C_{T,t}} \right)^{\frac{1}{\xi}} - \frac{1}{\xi} C_{T,t}^{-1} \right),$$

which allows us to characterize the absolute risk aversion as

$$\frac{-u_{11,t}}{u_{1,t}} = \frac{\sigma\xi - 1}{\xi} \alpha C_t^{\frac{1-\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}} + \frac{1}{\xi} C_{T,t}^{-1}.$$

Derivation of the Price Index We start by rewriting the intratemporal optimality condition as

$$C_{T,t} = \left(\frac{\alpha}{1-\alpha} \right)^{\xi} C_{N,t} \mathcal{E}_t^{-\xi}.$$

Defining the price index P_t as the price of aggregate consumption good C_t allows us to then write

$$\left(\frac{\alpha}{1-\alpha} \right)^{\xi} C_{N,t} \mathcal{E}_t^{1-\xi} + C_{N,t} = P_t C_t \Leftrightarrow C_{N,t} = \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha} \right)^{\xi} \mathcal{E}_t^{1-\xi} + 1 \right]},$$

and similarly

$$C_{T,t} \mathcal{E}_t + C_{T,t} \left(\frac{\alpha}{1-\alpha} \right)^{-\xi} \mathcal{E}_t^{\xi} = P_t C_t \Leftrightarrow C_{T,t} = \left(\frac{\alpha}{1-\alpha} \right)^{\xi} \mathcal{E}_t^{-\xi} \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha} \right)^{\xi} \mathcal{E}_t^{1-\xi} + 1 \right]}.$$

Next, plugging these expressions into the consumption aggregator we obtain

$$\begin{aligned} C_t &= \left[\alpha \left(C_{T,t} \right)^{\frac{\xi-1}{\xi}} + (1-\alpha) \left(C_{N,t} \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \\ &= \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha} \right)^{\xi} \mathcal{E}_t^{1-\xi} + 1 \right]} \left[\left(\frac{\alpha}{1-\alpha} \right)^{\xi} (1-\alpha) \mathcal{E}_t^{1-\xi} + (1-\alpha) \right]^{\frac{\xi}{\xi-1}} \\ &= P_t C_t \left[\alpha^{\xi} \mathcal{E}_t^{1-\xi} + (1-\alpha)^{\xi} \right]^{\frac{1}{\xi-1}}, \end{aligned}$$

which implies that the price index is given by

$$P_t = \left[\alpha^{\xi} \mathcal{E}_t^{1-\xi} + (1-\alpha)^{\xi} \right]^{\frac{1}{1-\xi}}.$$

Derivation of the Real Exchange Rate Define the real exchange rate as

$$\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t}.$$

Using the expression for the domestic price index and the fact that the price level abroad is normalized to unity, we obtain the following expression for the real exchange rate

$$\mathcal{Q}_t = \left[\alpha^{\xi} + (1-\alpha)^{\xi} \left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}.$$

B Calibration Details

Observed FXI regime While we do not know the exact FXI reaction function of the BNM, we can account for their FXI policy in the model by treating the observed FX interventions $\hat{B}_{M,t}$ as an exogenous process similar to portfolio outflows $\hat{B}_{P,t}$. For this purpose, we define a new variable that is the sum of portfolio outflows and FX interventions $\hat{B}_{PM,t} = \hat{B}_{P,t} + \hat{B}_{M,t}$ and replace $B_{P,t}$ by $B_{PM,t}$ as the third exogenous state variable. We can make this simplification since it implies that the only difference between $B_{P,t}$ and $B_{M,t}$ – namely that $B_{P,t}$ is exogenous while $B_{M,t}$ is a policy variable – then ceases to exist. Given the changed exogenous state variables, we estimate an AR(1) process $\tilde{\mathbf{s}}_t = \tilde{\rho}\tilde{\mathbf{s}}_{t-1} + \tilde{\varepsilon}_t$ where $\mathbf{s}_t = \left[\log Y_{T,t}, \log Y_{N,t}, \sinh^{-1}\left(B_{PM,t} - \overline{B}_{PM}\right)\right]'$ and $\overline{B}_{PM} = 0.24$. The error term $\tilde{\varepsilon}_t = [\tilde{\varepsilon}_{T,t}, \tilde{\varepsilon}_{N,t}, \tilde{\varepsilon}_{PM,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\tilde{\mathbf{V}}$ and $\tilde{\rho}$ is a 3×3 matrix consisting of the autocorrelation terms

$$\tilde{\mathbf{V}} = \begin{bmatrix} 0.0005258 & 0.0005685 & -0.000789 \\ 0.0005685 & 0.0008582 & -0.002274 \\ -0.000789 & -0.002274 & 0.1716685 \end{bmatrix} \quad \tilde{\rho} = \begin{bmatrix} 0.829771 & -0.414713 & -0.024469 \\ 0.220326 & 0.2561583 & -0.031291 \\ -1.15058 & -0.708053 & 0.441873 \end{bmatrix}.$$

In the data, FX interventions are negatively correlated with portfolio outflows at $\sigma_{B_P^*, B_M^*} = -0.3077$, which is directionally consistent with the optimal policy.

Calibration of the subjective discount factor β In the specification with an exogenous state process that accounts for the observed FXI regime (see above), we choose β such that the model matches the mean of the observed net foreign asset position as a percentage of annual GDP, which is 2%. The value of the subjective discount factor that matches this moment is $\beta = 0.9871$.

Calibration of financiers' risk aversion ω The target for the calibration of financiers' risk aversion is an average FX market depth of $\omega\overline{\sigma}^2 = 0.05$ in the observed economy. This condition holds for $\omega = 28$. Note that changing the financiers' risk aversion not only has implications for the endogenous variables in the model, but is also associated with a change in the estimated (exogenous) portfolio outflow process $B^*_{P,t}$, since ω enters the risk sharing wedge that we use to calculate these flows. As is evident from (15), higher risk aversion ω makes FX markets more shallow, magnifying the effect of portfolio outflows on the risk sharing wedge. Therefore, a given level of fluctuations in the risk sharing wedge can either be explained by more volatile portfolio outflows and deeper FX markets or less volatile portfolio outflows and more shallow FX markets.

C Simulation Details

Derivation of the Consumption Equivalence Letting κ be the additional fraction of consumption that households in the benchmark economy b will have to receive to make them indifferent to moving to an economy with the alternative policy p, we have

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left((1+\kappa)C_t^b\right)\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left(C_t^p\right)\right] \\
\Leftrightarrow &\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left((1+\kappa)C_t^b\right)^{1-\sigma}\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^p\right)^{1-\sigma}\right] \\
\Leftrightarrow &(1+\kappa)^{1-\sigma} \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^b\right)^{1-\sigma}\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^p\right)^{1-\sigma}\right].
\end{aligned}$$

Next, using

$$V_i\left(B^*, S\right) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[u\left(C_t^i\right)\right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{1}{1-\sigma}\left(C_t^i\right)^{1-\sigma}\right]$$

for $i \in \{b, p\}$ we obtain

$$\kappa = \left(\frac{V_{ap}\left(B^*, S\right)}{V_b\left(B^*, S\right)}\right)^{\frac{1}{1-\sigma}} - 1.$$

We weigh the value functions with the stationary distribution of the exogenous state process

$$\tilde{V}_i\left(B^*\right) = \sum_{S\in\mathcal{S}} \psi_S V_i\left(B^*, S\right),$$

where $\mathcal{S}$ is the set of all exogenous states S and ψ_S is the stationary probability of state S, with $\sum_{S\in\mathcal{S}} \psi_S = 1$. Finally, we impose the steady state net foreign asset position of the benchmark economy $B^* = \overline{B}^*$ as the initial condition to arrive at

$$\kappa = \left(\frac{\tilde{V}_{ap}\left(\overline{B}^*, \mathcal{S}\right)}{\tilde{V}_b\left(\overline{B}^*, \mathcal{S}\right)} \right)^{\frac{1}{1-\sigma}} - 1.$$

Solving for the first-best Given exogenous states $S = \{Y_T, Y_N, B_P^*\}$, the unconstrained planner solves

$$V(B^*, S) = \max_{B^{*\prime}} \left\{ u(R^* B^* + Y_T - B^{*\prime}, Y_N) + \beta \mathbb{E}_S V(B^{*'}, S') \right\},$$

subject to the market clearing conditions and a lower bound on bond holdings $\underline{B}^*$

$$\underline{B}^* \leq B^{*\prime} \leq R^* B^* + Y_T.$$

Note that we also impose the same lower bound $\underline{B}^*$ when computing all other policy regimes, but it is never binding in these cases since the economy is already subject to a stricter implicit borrowing limit associated with the lower bound on FX reserves. However, in the first-best, the latter constraint does not exist since the central bank is allowed to take a negative position in foreign currency bonds.

Additional Simulations Figure C.1 presents the paths of selected macroeconomic variables in an average episode of a fall in tradable endowment.

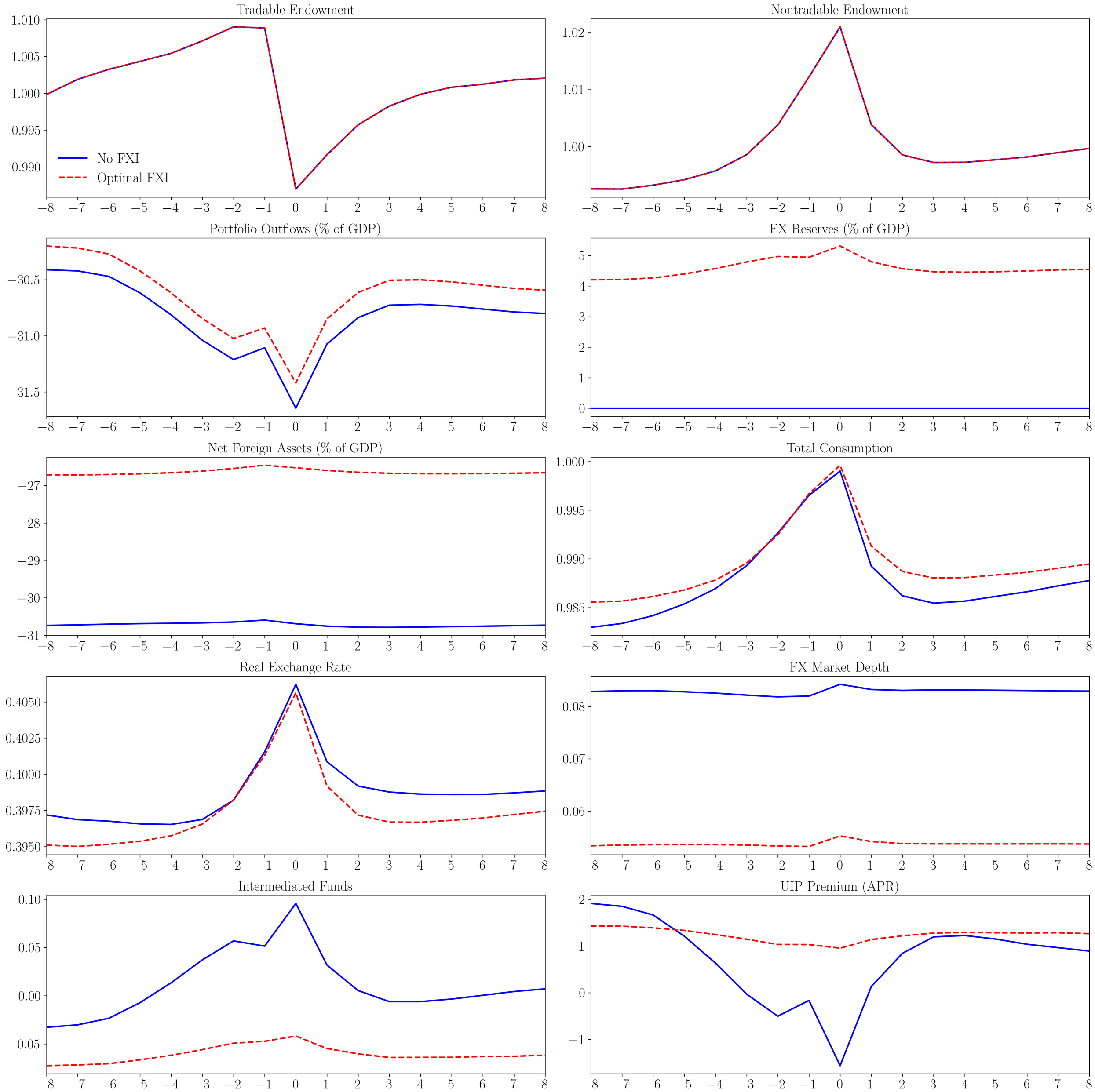

Figure C.1: Episodes of negative endowment shocks in the tradable sector

Notes: The figure depicts the average model response in episodes in which tradable endowment decreases relative to nontradable endowment. One period corresponds to one quarter and period 0 coincides with the tradable endowment trough.

Figure C.2 compares outcomes during the capital outflow episode considered in Figure 5 for the policies discussed in Section 5.

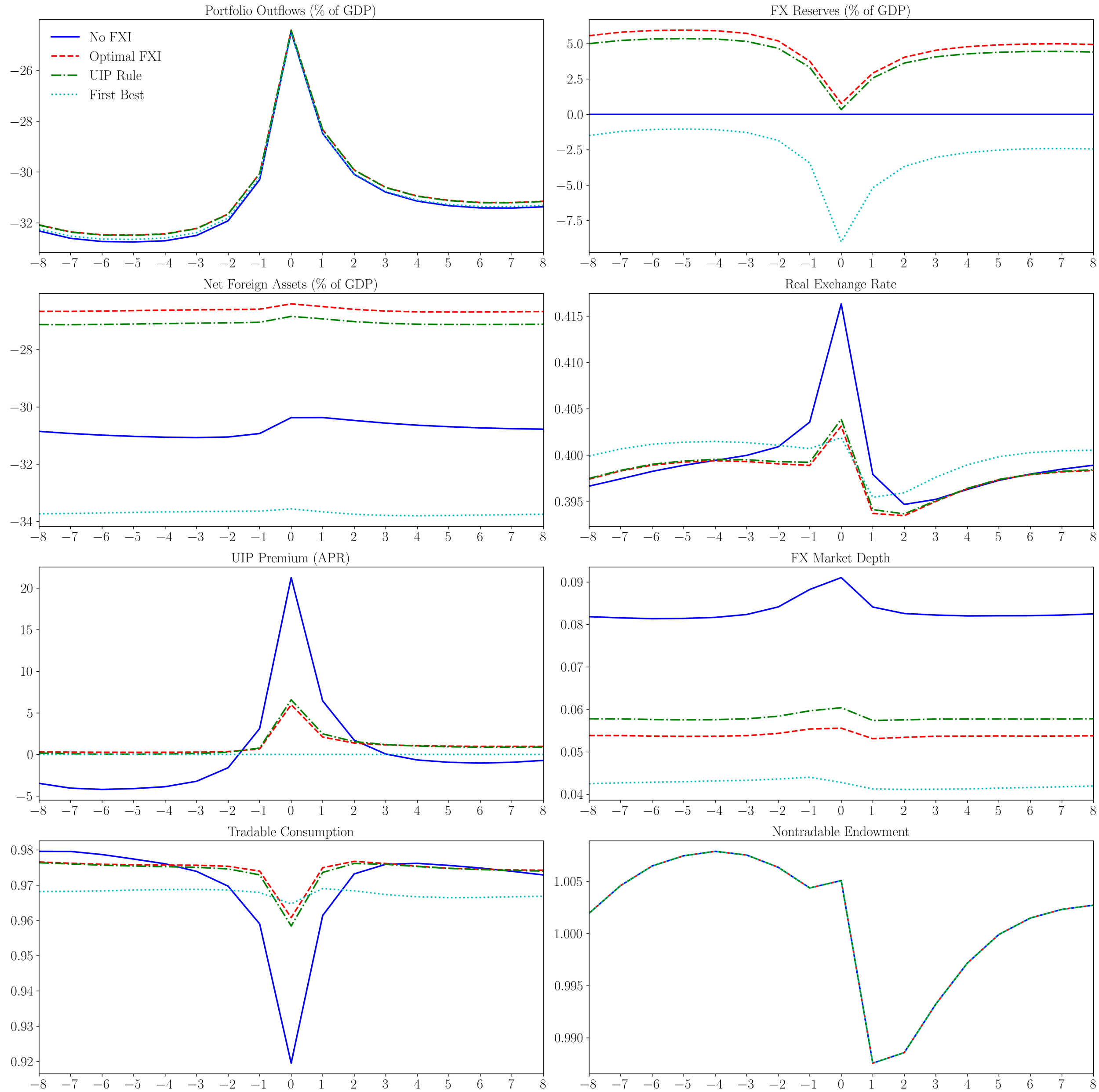

Figure C.2: Episodes of portfolio outflows under different policy regimes

Notes: The figure depicts the average model response in episodes of portfolio outflows for different policy regimes. One period corresponds to one quarter and period 0 coincides with the peak outflow.